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Trading with a common agent under complete information: A characterization of Nash equilibria

Gabriella Chiesa, Vincenzo Denicolò *

Department of Economics, University of Bologna, Piazza Scaravilli 2, I-40126 Bologna, Italy

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Abstract

We analyze an abstract model of trading where N principals submit quantity-payment schedules that describe the contracts they offer to an agent, and the agent then chooses how much to trade with every principal. This represents a special class of common agency games with complete information. We study all the subgame perfect Nash equilibria of these games, not only truthful ones, providing a complete characterization of equilibrium payoffs. In particular, we show that the equilibrium that is Pareto-dominant for the principals is not truthful when there are more than two of them. We also provide a partial characterization of equilibrium strategies.

JEL classification: C72; D43; D44

Keywords: Common agency; Supply schedules; Truthful equilibrium; Minimum rent equilibrium

1. Introduction

The theory of common agency applies to situations in which an agent makes a choice that affects N principals, each of whom tries to influence the agent's choice by offering payments contingent on it. This paper deals with a special class of complete information common agency games, those in which the agent's choice is an N-dimensional vector specifying the amount of a good to be traded with each principal. That is, principals first submit quantity-payment schedules describing the contracts they offer to the agent, and the agent then chooses how much to trade with every principal.

* Corresponding author.
E-mail address: vincenzo.denicolo@unibo.it (V. Denicolò).

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This abstract model of trading is flexible enough to encompass a wide variety of applications in economics. One, a natural extension of the standard model of Bertrand competition, is to the case of oligopolistic price discrimination, where the agent is a consumer who purchases a good from various firms that compete in non-linear prices (Spence [25], Spulber [26], Bhaskar and To [5]). Retaining the postulate that the agent *purchases* the good from the principals, other scenarios come readily to mind, such as vertical relationships (where several upstream firms share a common downstream agent that distributes the product),[1] split-award procurement (where a sponsor procures a good from various suppliers),[2] and markets for electricity or other intermediate goods (where a large buyer purchases from various firms).[3] The converse case in which the agent *sells* the good to the principals, first studied by Stole [24] and Martimort [14], can apply to multi-unit package auctions (where buyers bid for variable amounts of a divisible good),[4] or the regulation of multinational enterprises by agencies in several countries (where each national agency offers the multinational a payment conditional on the quantity sold in its country, as in Calzolari [7]).

In all of these applications, one would like to determine the trades that occur, the payments that are made, and the structure of the offers that the principals submit in equilibrium. While the quantities traded determine the equilibrium allocation, the payments determine the distribution of the gains from trade; they also determine players' incentives (in an earlier stage) to invest in research, bear fixed entry costs, or make other kinds of investments that may affect those gains. Finally, the structure of equilibrium strategies determines the computational complexity of the equilibria.

These questions have been resolved for the special case in which principals are assumed to submit truthful offers.[5] With truthful strategies, the equilibrium allocation is efficient (Bernheim and Whinston [4]), each principal obtains his marginal contribution to social surplus (Bergemann and Välimäki [3]), and equilibrium offers are, by definition, truthful.[6] However, much less is known for the case of unrestricted offers. The literature has shown that under suitable regularity

[1] See O'Brien and Shaffer [19,20]. This case is complementary to that analyzed by Martimort and Stole [16] and Segal and Whinston [23], where competing retailers distribute the output of a common manufacturer. With competing retailers there are *direct* externalities between principals, since the price obtained by a retailer depends on the output sold by the others. Our model, by contrast, has only *contractual* externalities.

[2] See Anton and Yao [1], who assume that the sponsor is committed to purchase a pre-specified amount of the good and can only choose how to split the procurement between the suppliers. In our model, by contrast, the sponsor can choose the total amount to be purchased after suppliers have submitted their offers.

[3] Electricity markets are often modeled using the notion of supply function equilibrium proposed by Klemperer and Meyer [10]. In this approach, buyers are price-takers and the unique equilibrium price is determined by equating aggregate supply and demand. In our common agency model, by contrast, a centralized buyer exercises market power by maximizing along each principal's "supply function," and principals trade at personalized prices. Which model is more realistic depends on the institutional details of actual electricity markets, which vary considerably from country to country and over time.

[4] The auction literature (see e.g. Krishna and Tranaes [12] and Milgrom [18]) has typically posited that the seller is committed to sell a pre-specified total amount, but Milgrom [18] argues the importance of the case of an uncommitted seller, which corresponds to our common agency model.

[5] Generally speaking, a strategy is said to be truthful relative to a given action if it truly, and for all cases, reflects the principals' marginal preferences for another action relative to the given action. This notion was originally developed for the case of public common agency, but it can be extended immediately to the present framework of private common agency with no direct externalities. In such a framework, truthfulness means that each principal can ask for payments that differ from his true valuations of the proposed trades only by a constant.

[6] These properties of truthful equilibria actually hold in a larger class of common agency models of which ours is a special case.