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Repeated games with one-memory ☆

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Abstract

We study the extent to which equilibrium payoffs of discounted repeated games can be obtained by 1-memory strategies. We establish the following in games with perfect (rich) action spaces: First, when the players are sufficiently patient, the subgame perfect Folk Theorem holds with 1-memory. Second, for arbitrary level of discounting, all strictly enforceable subgame perfect equilibrium payoffs can be approximately supported with 1-memory if the number of players exceeds two. Furthermore, in this case all subgame perfect equilibrium payoffs can be approximately supported by an ε-equilibrium with 1-memory. In two-player games, the same set of results hold if an additional restriction is assumed: Players must have common punishments. Finally, to illustrate the role of our assumptions, we present robust examples of games in which there is a subgame perfect equilibrium payoff profile that cannot be obtained with 1-memory. Thus, our results are the best that can be hoped for.

JEL classification: C72; C73; C79

☆ This is a revised version of Chapter 2 in Barlo [M. Barlo, Essays in game theory, PhD thesis, University of Minnesota, 2003], and of Sabourian [H. Sabourian, The Folk Theorem of Repeated Games with Bounded Recall (One-Period) Memory, Economic Theory Discussion Paper, vol. 143, University of Cambridge, 1989]. We thank Beth Allen, Pedro Amaral, Kemal Badur, Kevin Hasker, Ehud Kalai, Narayana Kocherlakota, Andy McLennan, Han Özsöylev, Aldo Rustichini, Jan Werner, an associate editor, an anonymous referee and participants at the Applied Microeconomics Workshop and Mathematical Economics Workshop at the University of Minnesota, the Games 2002 conference, the Mid-West Mathematical Economics 2002 meeting, the SAET 2003 conference, the ASSET 2003 and 2006 conferences, the SED 2006 conference, and the Winter 2007 North American Meeting of the Econometric Society for helpful comments and suggestions. Any remaining errors are ours.
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1. Introduction

The (subgame perfect) Folk Theorem of complete information repeated games states that any individually rational payoff can be sustained as a subgame perfect equilibrium (SPE) if the players are sufficiently patient—see [12] and [3]. Such a multiplicity of equilibria arises because in repeated games at any stage each player can condition his behavior on the past behavior of all the players. Such long memories are clearly unreasonable.

Many papers have modeled memory in repeated games by considering strategies that recall only a finite number past periods (see Section 2). Following this approach, in this paper, we restrict the strategies to those that depend only on what has happened in the previous period. We refer to such behavior by 1-*memory strategies*. (In this paper, memory refers to the number of past periods the players can recall; in the literature such memory limitations are also known as bounded recall.)

As suggested by Aumann [2], it is reasonable to expect that the extensive multiplicity of equilibria described by the Folk Theorem may be reduced by restricting players to limited memory strategies. In contrast, we show that if the set of actions in the stage game is sufficiently "rich" (i.e., it has a large number of actions), the Folk Theorem continues to hold with 1-*memory strategies*.

Notice that except for stationary strategies (that take the same action following every history), 1-memory strategies form, in terms of period recall, the simplest class of repeated game strategies. Despite being simple and highly restrictive in their dependence on the past, they are, in games with rich action spaces, able to generate a large set of equilibria. This is in sharp contrast to the case of stationary strategies, which can only implement outcomes that consist of repetitions of Nash equilibria of the stage game. Thus, it is surprising that, by increasing players' memory from zero to one period, the equilibrium set expands so significantly.

The richness of the set of actions assumption is critical in establishing the Folk Theorem (and the other results in this paper) with one period memory. Before discussing the fundamental role of this assumption, note that the Folk Theorem provides a characterization of the equilibrium set when the players discount the future by arbitrarily small amounts. But even when players are impatient, equilibrium strategies often require them to remember distant pasts. Therefore, we also ask whether or not, for any *arbitrary* level of discounting, we can obtain all SPE payoffs with 1-memory strategies, if the action sets are sufficiently rich.

For an arbitrary level of discounting, Abreu [1] showed that any SPE payoff profile can be supported by a *simple* strategy profile. We define a strictly enforceable SPE payoff profile as one that can be sustained by a simple strategy profile with the property that the players *strictly* prefer to follow the associated simple strategy at every subgame. Then our main result here establishes that if the action spaces are rich (formally, we assume that they are perfect sets[1]) and the number of players exceeds two, then any strictly enforceable SPE can be approximated by a 1-memory SPE. We also show that the same result holds with two players if, in addition, the equilibrium

[1] A subset of a topological space X is a *perfect* set if X is equal to the set of all limit points of X, or equivalently, X is a closed set with no isolated points. Therefore, perfect subsets of locally compact spaces are uncountable.