Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 144 (2009) 337–353
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Interim efficient allocations under uncertainty ☆

Atsushi Kajii [a,*], Takashi Ui [b]

[a] *Institute of Economic Research, Kyoto University, Japan*
[b] *Faculty of Economics, Yokohama National University, Japan*

Received 1 February 2006; final version received 12 September 2007; accepted 14 May 2008

Available online 1 July 2008

Abstract

This paper considers an exchange economy under uncertainty with asymmetric information. Uncertainty is represented by multiple priors and posteriors of agents who have either Bewley's incomplete preferences or Gilboa–Schmeidler's maximin expected utility preferences. The main results characterize interim efficient allocations under uncertainty; that is, they provide conditions on the sets of posteriors, thus implicitly on the way how agents update the sets of priors, for non-existence of a trade which makes all agents better off at any realization of private information. For agents with the incomplete preferences, the condition is necessary and sufficient, but for agents with the maximin expected utility preferences, the condition is sufficient only. A couple of necessary conditions for the latter case are provided.

JEL classification: D81; D82; D84

Keywords: Multiple priors; Interim efficiency; No trade; Dynamic consistency; Rectangular prior set

1. Introduction

This paper considers an exchange economy under uncertainty with asymmetric information. There are a finite number of states and in each state there is a single good. There are a finite number of agents and each agent has private information about the states. We model uncertainty by so-called multiple priors; that is, for each agent, uncertainty is represented by sets of priors

☆ This paper is an extended version of the paper entitled "Trade with Heterogeneous Multiple Priors" which is circulated as CAEA Discussion Paper No. 096.

* Corresponding author at: Kyoto University, Institute of Economic Research, Yoshida-Honcho, Sakyo-ku, Kyoto 606-8501, Japan.
E-mail addresses: kajii@kier.kyoto-u.ac.jp (A. Kajii), oui@ynu.ac.jp (T. Ui).

0022-0531/$ – see front matter
doi:10.1016/j.jet.2008.05.006

and sets of posteriors. The good is evaluated by concave utility index functions, from which agents derive either Bewley's [4] incomplete preferences (BI-preferences for short) or Gilboa–Schmeidler's [12] maximin expected utility preferences (MEU-preferences for short).

Prior sets induce preferences in the ex ante stage (before the receipt of private information) and posterior sets induce preferences in the interim stage (after the receipt of private information). An allocation is *ex ante efficient* if there is no feasible trade which makes all agents better off in the ex ante sage. Bewley [3] and Rigotti and Shannon [23] characterized ex ante efficient allocations by prior sets for agents with BI-preferences, and Billot et al. [2] characterized ex ante efficient allocations similarly for agents with MEU-preferences under no aggregate uncertainty.[1] An allocation is *interim efficient* if there is no feasible trade which makes all agents better off in the interim sage for any realization of private information. No attempt has been made to obtain a counterpart for interim efficient allocations as far as we are aware of.

The purpose of this paper is to provide characterizations of *interim efficient* allocations by posterior sets for agents with BI-preferences and MEU-preferences. The key concept in our characterizations is the *compatible prior set* of an agent, which is defined as the collection of all the probability distributions such that, for each piece of private information of the agent, the conditional probability distributions are in the corresponding posterior set of the agent. The compatible prior set of an agent coincides with the convex hull of all posteriors of the agent. The main results show the following: for agents with BI-preferences, an allocation is interim efficient if and only if it is ex ante efficient for agents possessing their compatible prior sets as their own prior sets; and for agents with MEU-preferences, an allocation is interim efficient if the same condition holds, but not vice versa. Thus, ex ante efficiency with respect to the compatible prior sets is necessary and sufficient for interim efficiency for the former case, but it is sufficient only for the latter case. To obtain a sharper result for the latter case, we restrict our attention to a limited class of allocations and provide a couple of necessary conditions for interim efficiency.

In the standard Bayesian models, Morris [21] and Feinberg [11] provided a characterization of interim efficient allocations,[2] which is closely related to the agreement theorem of Aumann [1]. The agreement theorem in this context asserts that if agents with linear utility index functions have a common prior, then an allocation is interim efficient. The result of Morris [21] and Feinberg [11] implies the converse: if an allocation is interim efficient for agents with linear utility index functions, then there is a prior which induces all the agents' posteriors; that is, it appears as if they share a fictitious common prior. Our results have the corresponding implication when utility index functions are linear; that is, for agents with BI-preferences, an allocation is interim efficient if and only if the compatible prior sets of all agents have a non-empty intersection, whose element is interpreted as a fictitious common prior.

Characterizations of interim efficient allocations are important in the context of the no trade theorem [20]: it asserts that any ex ante efficient allocation is interim efficient, as interpreted by Holmström and Myerson [17], and thus purely speculative trade is impossible. A simple and intuitive explanation for the no trade theorem is that agents in the standard Bayesian models are dynamically consistent. We show that an analogous but not identical explanation can be given for the multiple priors models, as follows. By combining the characterization of ex ante efficient allocations and that of interim efficient allocations, we can obtain a necessary and sufficient condition for any ex ante efficient allocation to be interim efficient for agents with BI-preferences.[3]

[1] Rigotti et al. [24] generalized this result.

[2] See also Samet [26] and Ng [22].

[3] On the no trade theorem in more general non-expected utility models, see Dow et al. [7], Ma [19], and Halevy [15].