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Notes, Comments, and Letters to the Editor

Decentralized trade, random utility and the evolution of social welfare

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Abstract

We study decentralized trade processes in general exchange economies and house allocation problems with and without money. The processes are affected by persistent random shocks stemming from agents' maximization of random utility. By imposing structure on the utility noise term—logit distribution—one is able to calculate exactly the stationary distribution of the perturbed Markov process for any level of noise. We show that the stationary distribution places the largest probability on the maximizers of weighted sums of the agents' (intrinsic) utilities, and this probability tends to 1 as noise vanishes.

JEL classification: C79; D51; D71

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1. Introduction

This paper considers the allocation of indivisible durable goods through decentralized trading processes. A simple example is the allocation of N offices among N students. Even if the size of the problem, N, is relatively small, the number of possible allocations can be quite large. With 10 students and offices, the number of allocations is about 3.6 million. We examine how

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successful decentralized trading processes, both with and without money, are in solving those complex combinatorial problems.

We consider a situation where agents randomly meet over time. When a group of agents meet, they exchange their goods in the following simple way. First, a new allocation for them is randomly proposed, and it is accepted if it provides a higher utility for all of them. Otherwise, the agents continue to hold their endowments. [1] When they assess the proposed allocation, we assume that their utility is affected by random shocks. The shocks can be interpreted as mistakes, or transitory changes in tastes attributed to noise; see, for example, the related notion of quantal response equilibrium in behavioral game theory [8].

Incorporating random terms in utility functions has been found to be quite useful in econometric studies of discrete choice problems and behavioral game theory, and we employ one of the leading specifications, the *logit model*, for the distribution of the noise term. Thanks to the special structure of the model, we obtain the closed form solution of the stationary distribution, for *any level of noise*. In this respect, our work is built on the literature pioneered by Blume [3,4], who identified a set of conditions which enables one to derive the closed form stationary distribution under logit noise. Our technical contribution is to show that a similar closed form can be obtained in a wider class of models, even when Blume's conditions are not satisfied. [2]

This approach contrasts with the traditional long-run stochastic stability methodology (see [6,11]). The method identifies those states—allocations—in which the economy spends most of its time in the long run, when the noise in the system is made *negligible*. Negligible noise implies a fairly long waiting time to see the long run effects. The present paper, in contrast, allows us to characterize the stationary distribution *for any level of noise*. Specifically, we show that, for any level of noise, the states that maximize a weighted sum of the agents' intrinsic utilities receive the largest probability in the stationary distribution.

Our result sheds light on the previous contribution by Ben-Shoham et al. [2]. They considered house allocation problems and found that, with vanishing noise, *the minimum envy allocation* is selected when serious mistakes are less likely. An agent's envy level is the number of other agents who have better houses, and the minimum envy allocation is the one that minimizes the aggregate envy level. We show that this somewhat mysterious result can be derived from a more general principle, namely, that evolutionary dynamics with logit noise maximize the aggregate utility level. [3]

Our results imply, in particular, that the most likely state is efficient. Note that, with no noise, our exchange processes may be stuck on an inefficient state. For example, when only *bilateral trades* are possible, the society may be stuck on an inefficient state where there is *no double coincidence of wants*. [4] In this respect, our decentralized trading processes resemble the algorithms to solve combinatorial optimization problems, where the process may get stuck at a local maximum. For the latter problems, random search algorithms, notably simulated annealing methods (see [1]) have been found quite effective. Just like randomness in simulated annealing helps to escape from

[1] Hence, agents in our model act myopically. Either bounded rationality or the arrival rate of trading opportunities being low, relative to the discount factor, might justify this.

[2] Section 4 provides a detailed discussion.

[3] See Section 4 again for details.

[4] Ben-Shoham et al. [2] showed that an *in*efficient state can be stochastically stable, when all mistakes are equally likely. Hence adding noise does not always help escape from an inefficient state. Our model provides a set of sufficient conditions for the noise term to knock out inefficient states.