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Equilibrium in the two-player, k-double auction with affiliated private values

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Abstract

We present sufficient conditions for the existence of an increasing equilibrium in the two-player, k-double auction with affiliated private values. In the case $k \in \{0, 1\}$ there exists a unique equilibrium in non-dominated strategies. Applying this equilibrium, we provide a detailed example illustrating how dependence level affects strategic bidding. In the case $k \in (0, 1)$ we prove existence of a continuum of strictly increasing equilibria, and illustrate them using an example. Furthermore, we show that equilibria in the case of independent private values are pointwise limits of equilibria with strictly affiliated private values.

JEL classification: D44

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1. Introduction

The k-double auction is, perhaps, the simplest strategic model of supply and demand that incorporates two-sided asymmetric information. This model has proven itself to be a useful tool for investigating the effect of asymmetric information upon trade. In this mechanism, buyers and sellers submit sealed bids to a market organizer. The organizer creates demand and supply schedules using the individual bids and offers, and finds a market clearing price. This price is determined by an exogenous parameter $k \in [0, 1]$. Buyers, who submitted bids higher than the market clearing price trade with sellers, who submitted offers lower than the market clearing

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price. This kind of trading mechanism is used throughout the world as an opening stage for trade in stock exchanges such as the NYSE, London, Toronto, and others.

The two-player, k-double auction was first introduced by Chatterjee and Samuelson [2]. This mechanism has been extensively explored later on by Williams [23], Satterthwaite and Williams [20], and Leininger et al. [12]. These papers show existence of different kinds of equilibria, and investigate their efficiency attributes. All of these papers assume that the valuations of the buyer and the seller are independent from one another. Other papers show the implications of increasing the level of competition in this mechanism on market efficiency. These include: Wilson [25], Satterthwaite and Williams [21,22], Williams [24], Rustichini et al. [19], Zacharias and Williams [26], and Kadan [9]. These papers assume independent valuations as well.

Existence of an increasing equilibrium in the more realistic case of dependent values has long been an open question. Recently, several papers made major contributions to this area. Jackson and Swinkels [8] show existence of equilibrium in distributional strategies in a large class of private and dependent value double auctions. They allow for any number of buyers and sellers and do not require symmetry. However, they do not obtain monotonicity, nor do they consider the case of one buyer and one seller. Reny and Perry [18] analyze a double auction in the case $k \in \{0, 1\}$. In their model, valuations have both a private and a common component. They show existence of a non-decreasing equilibrium given that the number of traders is sufficiently large, and given that bids and offers are restricted to a discrete grid. Fudenberg et al. [6] show existence of a pure strategy, increasing, symmetric equilibrium in large private, and dependent value double auctions. The idea is that when the number of traders is sufficiently large, bids tend to truth telling, and therefore become strictly increasing. [1]

While these papers provide answers to the existence question under many circumstances, there still are some open questions left. First, does there exist a pure strategy, increasing equilibrium when the number of traders is small, and values are dependent? In this case, strategic behavior is profound and it might destroy the monotonicity of best response correspondences. Equilibria in strictly increasing pure strategies have been in the focus of much of the research on double auctions and one-sided auctions. Such equilibria have been utilized for empirical, experimental, and practical purposes since they provide easy and intuitive predictions. Therefore, it is important to know whether those equilibria are just an artifact of the independence assumption.

Second, what is the nature of equilibria with dependent values, and how does the level of dependence affect the bidding strategies? All the papers proving existence in dependent value double auctions use powerful fixed point arguments. These arguments, however, limit the analysis of equilibrium and do not yield comparative statics with respect to the level of dependence.

Finally, are equilibria in the independent private value case the limits of equilibria of double actions with dependent values? Dependence seems to be the norm in the real world while independence is a convenient simplifying assumption. Thus, it is important to know that the equilibria identified in the independent value case are robust to small deviations from independence. Notice that continuity with respect to dependence level is not obvious. Crémer and McLean [3,4], Gresik [7], and McAfee and Reny [14] show that efficient mechanism design is possible with dependent values contrary to its impossibility in independent private value frameworks. [2] In contrast, Kosmopoulou and Williams [10] show that the robustness of the independent private value framework is restored if the magnitude of the monetary transfers in the mechanism design is bounded. In an equilibrium framework, lower hemi-continuity is not obvious either. Even when

[1] Cripps and Swinkels [5] show the convergence to efficiency of private value double auctions with dependent values.

[2] See Myerson and Satterthwaite [17] for the impossibility result.