Journal of Economics and Business 71 (2014) 68–89

Contents lists available at ScienceDirect

Journal of Economics and Business

Portfolio optimization in an upside potential and downside risk framework☆

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ARTICLE INFO

Article history:
Received 12 January 2011
Received in revised form 11 July 2013
Accepted 12 August 2013

Keywords:
Portfolio optimization
Von Neumann and Morganstern utility theory
Lower partial moment and upper partial moment optimization formulation
Four utility cases

ABSTRACT

The lower partial moment (LPM) has been the downside risk measure that is most commonly used in portfolio analysis. Its major disadvantage is that its underlying utility functions are linear above some target return. As a result, the upper partial moment (UPM)/lower partial moment (LPM) analysis has been suggested by Holthausen (1981. *American Economic Review*, *v71*(1), 182), Kang et al. (1996. *Journal of Economics and Business, v48*, 47), and Sortino et al. (1999. *Journal of Portfolio Management*, *v26*(1,Fall), 50) as a method of dealing with investor utility above the target return. Unfortunately, they only provide dominance rules rather than a portfolio selection methodology. This paper proposes a formulation of the UPM/LPM portfolio selection model and presents four utility case studies to illustrate its ability to generate a concave efficient frontier in the appropriate UPM/LPM space. This framework implements the full richness of economic utility theory be it [Friedman and Savage (1948). *Journal of Political Economy*, *56*, 279; Markowitz, H. (1952). *Journal of Political Economy*, *60*(2), 151; Von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior.* (3rd ed., 1953), Princeton University Press], and the prospect theory of (Kahneman and Tversky (1979). *Econometrica*, *47*(2), 263).

☆ The authors would like to thank D. Hillier, S. Brown, J. Estrada, T. Post, P. Van Vliet, F. Viole, anonymous referees and the Finance Seminar Series at Villanova University for their comments on earlier versions of this paper. All errors of course are the responsibility of the authors.

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http://dx.doi.org/10.1016/j.jeconbus.2013.08.001

The methods and techniques proposed in this paper are focused on the following computational issues with UPM/LPM optimization.

- Lack of positive semi-definite UPM and LPM matrices.
- Rank of matrix errors.
- Estimation errors.
- Endogenous and exogenous UPM and LPM matrices.

1. Introduction

The mean-lower partial moment (μ,LPM) model has been attractive to decision makers because it does not require any distributional assumptions and it is a necessary and sufficient condition for investors with various classes of Von Neumann and Morgenstern (1944) (hereafter, vNM) utility functions which is equivalent to expected utility-maximization under risk aversion.[1] Because it does not make any distributional assumption, it has been particularly useful in the management of derivative portfolios (Merriken, 1994; Huang, Srivastava, & Raatz, 2001; Pedersen, 2001; and Jarrow & Zhao, 2006).

However, LPM has traditionally been challenged by academic researchers because of the computational complexity of the asymmetric Co-LPM matrix used in μ-LPM portfolio analysis and the persistent belief that it is an ad-hoc method that is not grounded in capital market equilibrium theory and in expected utility maximization theory.[2]

A major challenge to the use of any portfolio theory formulation that does not use mean-variance analysis is by Markowitz (2010). His position is even with non-normal security distributions, the mean-variance criterion is still a useful approximation of the expected utility of the investor. In other words, any alternative to mean-variance portfolio theory has to rest on a solid foundation of utility theory. It is not sufficient for the portfolio framework to simply be a nonparametric approach. The UPM/LPM framework is powerful because it is a nonparametric approach and it implements the full richness of economic utility theory be it Friedman and Savage (1948), Markowitz (1952), Von Neumann and Morgenstern (1944), or the prospect theory of Kahneman and Tversky (1979). Markowitz (2010) does end up supporting the geometric mean-semivariance portfolio theory model in his paper because of its utility theory foundation. Semivariance is the only risk measure other than variance that is accorded any support by Markowitz (1959, 2010).

While our focus is not on LPM and capital market theory, the discussion in Hogan and Warren (1974), Bawa and Lindenberg (1977), Harlow and Rao (1989), Leland (1999), and Pedersen and Satchell (2002) makes it pretty clear that LPM is not an ad-hoc model that is ungrounded in capital market theory. We are interested in solving the computational complexities of the μ-LPM and its well-known utility maximization limitation of assuming a linear utility function above the target return.[3] By solving the μ-LPM computational problem, we are able to introduce the upper partial moment-lower partial moment (UPM/LPM) portfolio selection model which extends the expected utility maximization capabilities of the LPM model.

The paper continues with a discussion of mean-LPM and UPM/LPM portfolio analysis and their place in expected utility theory. Next, we offer a formulation for testing UPM/LPM portfolio optimization problems and discuss the historic issue of exogenous and endogenous LPM matrices. Next, four empirical problems are discussed which include: (1) Lack of positive semi-definite UPM and LPM matrices; (2) Rank of matrix errors; (3) Estimation errors; and (4) Endogenous and Exogenous UPM

[1] See Frowein (2000) for the necessary and sufficient conditions.

[2] See Grootveld and Hallerbach (1999) for a discussion of empirical issues and Pedersen and Satchell (2002) for a discussion of the theoretical foundation of μ-LPM in capital market theory.

[3] LPM utility functions are interested only in downside or below target return risk. It assumes a risk-neutral investor for above-target returns. See Fishburn (1977), Fishburn and Kochenberger (1979) and Kaplan and Siegel (1994a, 1994b).