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Properties of the utility function: A market-based analysis

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ABSTRACT

Using US market data, this paper sheds new empirical light on *properties of the utility function*. In particular, employing theoretical relations between Stochastic Discount Factors, state prices, and state probabilities, we are successful in recovering the following four *functions*: (i) Absolute Risk Aversion (ARA); (ii) Absolute Risk Tolerance (ART); (iii) Absolute Prudence (AP); and (iv) Absolute Temperance (AT). Our statistical analysis points out, unequivocally, that the ARA function is decreasing and convex, the ART function is convex, AT is greater than ARA, and the AP function is not decreasing. These empirical results are analyzed in light of established theory concerning, inter-alia, precautionary saving and prudence as well as the way risk attitudes are affected by the presence of "background risks" and by investors' investment horizon.

1. Introduction

Using US asset market data, this paper sheds new empirical light on properties of investors' von Neumann–Morgenstern (vNM) utility function. In particular, employing theoretical relations between Stochastic Discount Factors (SDFs), state prices, and physical state probabilities, we recover the following four *functions*: (i) Absolute Risk Aversion (ARA); (ii) Absolute Risk Tolerance (ART); (iii) Absolute Prudence (AP); and (iv) Absolute Temperance (AT).[2]

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[1] The research was performed while Ori was at the University of Haifa, Israel.

[2] He and Leland (1993), Wang (1993), Dybvig and Rogers (1997), and Cuoco and Zapatero (2000) discuss the recoverability of preferences from observed asset prices and an agent's consumption choice. Jackwerth (2000) is the first to use estimates of state prices and physical probabilities to recover *A* RA functions. We extend on that by characterizing additional properties of the *vNM utility* function, and relying on a more general approach for estimating state probabilities. Our approach is shown to have non-trivial consequences.

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An empirical characterization of the vNM utility function is of importance for a number of reasons. First, as is well known, risk aversion is expressed by the concavity of the utility function, with the Arrow–Pratt coefficient of ARA measuring its intensity—a more concave utility function induces stronger risk aversion (Pratt, 1964). Furthermore, Decreasing Absolute Risk Aversion (DARA) results in wealthier people being less reluctant to accept a given risk (Pratt, 1964) and is sufficient to guarantee the existence of a precautionary saving motive (i.e., future uncertainties reduce current consumption and increase current saving). In this paper, we use option price data and the resulting *state-by-state* estimates of the SDF to search for empirical evidence for the existence of DARA (as well as some other prominent preference-related phenomena, described below). Such a "state-dependent" approach sharply contrasts with ones based on consumption-based point estimate of risk aversion coefficients (e.g., Hansen & Singleton, 1982), which are evidently ill-suited to corroborate or refute DARA.[3]

Second, properties of the utility function for wealth – the ART function, the inverse of the ARA function – turn out to be decisive in determining the qualitative effect of investors' age on their risk-taking behavior. In particular, Gollier and Zeckhauser (2002) show that a necessary and sufficient condition for young investors to be less risk averse than old investors is the convexity of the ART function (see their "benchmark" case).[4] To the best of our knowledge, there is no empirical evidence on whether Absolute Risk Tolerance is concave or convex with respect to wealth, so in this paper we provide direct asset market evidence regarding the convexity of the ART function.

Third, it is well-known that if a precautionary saving motive (prudence) is to exist, the marginal utility function should be convex (Leland, 1968; Sandmo, 1970). The coefficient of AP (Kimball, 1990) measures the strength of the precautionary saving motive, such that a more convex marginal utility induces a stronger precautionary saving motive. Using different measures of uncertainty, estimation methods and datasets, Dynan (1993), Carroll (1994), Merrigan and Normandin (1996), and Ludvigson and Paxson (2001) provide point estimates (rather than functions) of the strength of the precautionary saving motive. As the strength of the proposed market approach is in providing state-by-state estimates of the SDF, we are able to estimate *state-dependent* coefficients of AP, that is, estimates of AP *functions*.

Finally, restrictions on investors' preferences affecting the willingness to accept a given risk when another independent, uninsurable "background risk" is added to total wealth are discussed.[5] Specifically, Kimball's (1993) *standard risk aversion* and Gollier and Pratt's (1996) *risk vulnerability*, differing in the restrictions imposed on the added background risk, are introduced.[6] In particular, (i) a necessary and sufficient condition for standard risk aversion, i.e., that an undesirable background risk reduces the optimal investment in a risky, independent prospect, is both DARA and Decreasing Absolute Prudence (DAP, that is, a precautionary saving motive declining with wealth); (ii) a sufficient condition for risk vulnerability is DARA and a convex ARA function[7]; and (iii) as found by Gollier and Pratt (1996), a necessary and sufficient condition for every small unfair background risk to induce more risk averse behavior with respect to an independent prospect ("local risk vulnerability") is that both the coefficient of AP and the coefficient of AT (Kimball, 1992) are larger than the coefficient of ARA.

The background risk problem is analyzed in light of the above-mentioned theoretical results. The estimated ARA and AP functions shed light on whether investors' vNM utility function is risk vulnerable/standard. Once again, since our methodology yields state-by-state estimates of the SDF, and since AT functions depend on derivatives of the SDF, we are successful in recovering (from observed prices), along with ARA and AP functions, repeated temporal state-dependent coefficients of AT, that

[3] Recently, Ogaki and Zhang (2000, 2001) used household level consumption data to find evidence for decreasing relative risk aversion.

[4] Guiso, Jappelli, and Terlizzese (1996), in a cross-section of Italian households find, quite surprisingly, that young people hold the smallest proportion of risky assets in their portfolios.

[5] Pratt and Zeckhauser (1987) provide the first analysis of the background risk problem, introducing the concept of "proper risk aversion."

[6] "Properness" (Pratt & Zeckhauser, 1987), "standardness" (Kimball, 1993), and "risk vulnerability" (Gollier & Pratt, 1996) deal with undesirable risks, expected marginal utility-increasing risks, and unfair (non-positive) risks, respectively. Eeckhoudt, Gollier, and Schlesinger (1996) discuss a more general set of changes in background risk. See Gollier (2001) for a textbook treatment.

[7] Note that standard utility functions are vulnerable to risk (see Gollier, 2001, chap. 9; Gollier & Pratt, 1996).