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Dynamic conditional correlation multiplicative error processes☆

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ABSTRACT

We introduce a dynamic model for multivariate processes of (non-negative) high-frequency trading variables revealing time-varying conditional variances and correlations. Modeling the variables' conditional mean processes using a multiplicative error model, we map the resulting residuals into a Gaussian domain using a copula-type transformation. Based on high-frequency volatility, cumulative trading volumes, trade counts and market depth of various stocks traded at the NYSE, we show that the proposed transformation is supported by the data and allows capturing (multivariate) dynamics in higher order moments. The latter are modeled using a DCC-GARCH specification. We suggest estimating the model by composite maximum likelihood which is sufficiently flexible to be applicable in high dimensions. Strong empirical evidence for time-varying conditional (co-)variances in trading processes supports the usefulness of the approach. Taking these higher-order dynamics explicitly into account significantly improves the goodness-of-fit and out-of-sample forecasts of the multiplicative error model.

1. Introduction

The modeling of intraday trading processes has become a major area in financial econometrics. This is particularly triggered by technological progress on financial markets, changing institutional structures in the trading landscape and a growing importance of intraday trading. The availability of financial data on the lowest possible aggregation level opens up the possibility to gain a deeper understanding of financial trading processes and to successfully manage trading risks, trading costs and intraday price risks.

This paper contributes to the literature on multivariate models for trading processes. We propose a model capturing trading dynamics not only in first conditional moments but also in conditional (co-)variances. The latter reflect the time-varying uncertainty inherent in intraday trading processes as well as dynamic correlation structures between key trading variables. The major idea is to map innovations in non-negative dynamic processes into a Gaussian domain using a copula-type transformation. The innovations stem from a vector multiplicative error model (VMEM) as proposed by Manganelli (2005) and Cipollini et al. (2007). The transformation of observations into a Gaussian domain allows identifying non-linear dependencies between trading variables and yields a natural separation of (multivariate) dynamics in first and second conditional moments. The latter are conveniently captured using dynamic conditional correlation (DCC) models as proposed by Engle (2002a). The proposed approach is sufficiently flexible to be applicable in high dimensions and can be extended in various directions.

Multiplicative error models (MEMs) — labeled according to Engle (2002b) — are workhorses for the modeling of dynamic processes of non-negative random variables, such as trading volumes, volatilities, trading intensities or market depth. The principle of decomposing a

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process into the product of its conditional mean and a positive-valued error term is well known in the literature and builds the backbone of the autoregressive conditional heteroscedasticity (ARCH) model by Engle (1982) and the stochastic volatility (SV) model introduced by Taylor (1982). In high-frequency econometrics, it has been put forward by Engle and Russell (1998) to model the dynamics of trade-to-trade durations and has been referred to as autoregressive conditional duration (ACD) model.[1]

A difficulty in the modeling of non-negative random variables is that typical distributions, such as the exponential distribution or generalizations thereof, imply a direct relationship between all moments. Accordingly, a MEM process implies that higher order (conditional) moments follow the same dynamics which, however, is not necessarily supported by the data. To address this problem, Ghysels et al. (1998) propose a two-factor model allowing to separate dynamics of the conditional mean and the conditional variance. Their principle is to rewrite an exponential model with gamma heterogeneity in terms of two Gaussian factors following a bi-variate dynamic process. Though this model accounts for features in trading variables which are not captured by a basic MEM, it imposes (partly restrictive) distributional assumptions and is hard to estimate. In a multivariate setting, the situation is even more complicate as not only conditional variance dynamics but also time-varying correlation structures have to be taken into account. However, finding a sufficiently flexible multivariate distribution defined on positive support is a difficult task. As discussed by Cipollini et al. (2007), a possible candidate is a multivariate gamma distribution which however imposes severe restrictions on the contemporaneous correlations between the variables.

This paper's contribution is to capture higher-order dependence structures using a Gaussian copula-type decomposition of dynamics. Capturing conditional mean dynamics using a VMEM specification, the resulting residuals serve as serially uncorrelated innovations whose multivariate distribution is mapped into a Gaussian domain. This mapping has two major advantages: first, it allows to straightforwardly link the individual marginal distributions to an appropriate joint distribution. Moreover, the imposed normality enables to naturally disentangle first and second conditional moments. Furthermore, the mapping into a Gaussian domain allows identifying non-linear (cross-)dependencies in trading processes which are not identifiable using a basic (linear) VMEM. The dynamics in resulting transformed innovations are naturally captured using (V)ARMA-GARCH and DCC-type specifications. This makes the model quite flexible and applicable in high dimensions. Accordingly, we suggest a composite maximum likelihood estimation procedure which is also feasible for high-dimensional processes.

We apply the model to 5-min squared mid-quote returns, cumulative trading volumes, trade counts as well as market depth of different stocks traded at the New York Stock Exchange (NYSE). We show that the normality-induced separation between first and second conditional moments is well supported by the data. It turns out that VMEM innovations still reveal substantial dependencies in higher moments which are only identifiable after the application of the Gaussian transformation. It turns out that the explicit consideration of these dependencies leads to a significantly better fit in terms of information criteria.[2]

Our study shows that trading variables are subject to time-varying conditional variances reflecting uncertainty in liquidity variables and volatility (so-called “liquidity risk” and “volatility risk”, respectively). The processes are quite persistent and reveal positive cross-dependencies. Hence, uncertainty in volatility and liquidity tends to spill over from one variable to another. Moreover, we show that conditional correlations between liquidity and volatility variables substantially vary over time. These insights are interesting from a microstructure and trading perspective as they allow identifying periods where connections between liquidity demand, liquidity supply and volatility are particularly high or low, respectively. Residual diagnostics show that the proposed approach explains the multivariate dynamics in trading processes clearly better than a basic (linear) VMEM specification. Moreover, it is shown that out-of-sample forecasts are improved.

The proposed dynamic conditional correlation MEM complements the existing literature on multiplicative error processes and the modeling of intraday trading. Various aspects which have been addressed in extant literature can be included in our approach. For instance, latent factor approaches in the spirit of Bauwens and Veredas (2004) and Hautsch (2008), component MEMs, as proposed by Brownlees et al. (2011) or Brownlees and Vannucci (2013), long memory dynamics, as put forward by Jasiak (1998) and Karanasos (2004), or regime-switching MEMs as in Zhang et al. (2001) or Meitz and Teräsvirta (2006) could be easily included in the basic (V)MEM specification. Likewise, the included DCC-GARCH component could be further extended by recent advances in the literature on multivariate GARCH models (see, e.g., Bauwens et al., 2006). Finally, our approach contributes to the empirical literature on dynamic copula models, see, e.g., Patton (2006), or on copula-based multivariate GARCH processes as suggested by Jondeau and Rockinger (2006), Lee and Long (2009) and Liu and Luger (2009). A study related to ours is Cipollini et al. (2007) who also suggest copulas in a (V)MEM setup. An important difference, however, is that in their context, the copula parameters are assumed to be constant, while in our context, remaining dynamics (after the transformation) are explicitly captured.

The remainder of the paper is organized as follows. Section 2 briefly reviews the basic vector multiplicative error model. Section 3 introduces the new copula-based approach. In Section 4, we provide an empirical application to the modeling of high-frequency trading processes. Finally, Section 5 concludes.

2. The basic multiplicative error model

Let $\{X_t\}$, $t = 1, \ldots, T$, denote a non-negative valued random process and let $\mathcal{F}_t$ define the information set up to time t. The basic univariate multiplicative error model (MEM), as introduced by Engle (2002b), is given by

$$X_t = \mu_t \varepsilon_t, \varepsilon_t | \mathcal{F}_{t-1} \sim \text{i.i.d.}\, D\left(1, \sigma^2\right),$$

[1] See, e.g., Hautsch (2012) for an overview.

[2] Of course, an obvious drawback of a Gaussian mapping is that it cannot appropriately capture heavy tails and tail dependence structures. Our empirical results, however, indicate that these effects are not very present in the given application, as the resulting Gaussianity is empirically supported.