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Portfolio optimization for heavy-tailed assets: Extreme Risk Index vs. Markowitz

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ABSTRACT

Using daily returns of the S&P 500 stocks from 2001 to 2011, we perform a backtesting study of the portfolio optimization strategy based on the Extreme Risk Index (ERI). This method uses multivariate extreme value theory to minimize the probability of large portfolio losses. With more than 400 stocks to choose from, our study seems to be the first application of extreme value techniques in portfolio management on a large scale. The primary aim of our investigation is the potential of ERI in practice. The performance of this strategy is benchmarked against the minimum variance portfolio and the equally weighted portfolio. These fundamental strategies are important benchmarks for large-scale applications. Our comparison includes annualized portfolio returns, maximal drawdowns, transaction costs, portfolio concentration, and asset diversity in the portfolio. In addition to that we study the impact of an alternative tail index estimator. Our results show that the ERI strategy significantly outperforms both the minimum-variance portfolio and the equally weighted portfolio on assets with heavy tails.

1. Introduction

In this paper we propose and test a portfolio optimization strategy that aims to improve the portfolio return by stabilizing the portfolio value. Minimizing the probability of large drawdowns, this strategy can help to retrieve the portfolio value as good as possible also in times of high risk in the markets. This intended performance is, of course, not a new aim in portfolio management, and it became even more vital since the default of Lehman Brothers in 2008. The following years of financial crisis have demonstrated that the technical progress of financial markets and their globalization have also brought up some new challenges. One of these challenges is the need for diversification strategies that account for strong drawdowns and increasing dependence of asset returns in crisis periods. This has raised the relevance of non-Gaussian models, tail dependence, and quantile based risk measures in portfolio optimization (Chollete et al., 2012; DeMiguel and Nogales, 2009; DeMiguel et al., 2009; Desmoulins-Lebeault and Kharoubi-Rakotomalalaé, 2012; DiTraglia and Gerlach, 2013; Doganoglu et al., 2007; Garlappi et al., 2007; He and Zhou, 2011; Hu and Kercheval, 2010; Hyung and de Vries, 2007; Mainik and Rüschendorf, 2010; Ortobelli et al., 2010; Rachev et al., 2005; Zhou, 2010).

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1.1. Developments in theory and practice of portfolio optimization

Since its introduction by Markowitz (1952), the mean–variance approach became the industry standard for asset allocation. However, this popularity also brought up several technical issues in practical applications, and there has been a large amount of further development addressing them.

One main direction of related research is dedicated to the impact of parameter uncertainty on the investment performance. The high sensitivity of the estimated mean–variance efficient portfolio to estimation errors in the underlying distribution parameters (expectations and covariances of asset returns) may lead to highly non-robust results. Barry (1974) and Chopra and Ziemba (1993) show the high sensitivity in particular when estimating the expected returns. Jorion (1985, 1986, 1991) and Jagannathan and Ma (2003) find that the pure minimum variance (MV) portfolio may outperform the mean–variance efficient portfolio.

Several approaches addressing the statistical challenge of parameter uncertainty have been suggested in the literature. These include the use of Bayesian and shrinkage estimators, shrinking the portfolios to some predetermined target which depends on combination of prior information with sample data (see, e.g., Jorion (1985, 1986)). Black and Litterman (1991) suggest Bayes estimation of means and covariances. However, their findings on the superiority of the Bayes/Stein procedure are not confirmed in some other studies like Fletcher and Leyffer (1994) and Fletcher (1997) and Grauer and Hakansson (1995). DeMiguel et al. (2009) and DeMiguel and Nogales (2009) investigate the potential advantage of robust optimization and shrinkage estimators. The resulting picture is, however, not completely clear, and it turns out that even robustified and optimized procedures in some cases fail to outperform simple heuristic strategies like the equally weighted portfolio.

Concerning robust asset allocation, Tütüncü and Koenig (2004) look for robust solutions that have the optimal worst-case performance, whereas Goldfarb and Iyengar (2003) choose worst-case estimators in a robust model framework that can be solved by linear programming. Herold and Maurer (2006) observe that even these more stable estimation methods only outperform simple strategies when combined with regression models for the expected return.

Another research direction includes several approaches to change the objective function in the optimization problem underlying the investment strategy. One of the issues addressed here is that quantification of risk by variance does not distinguish between gains and losses. Hence, to avoid wrong conclusions for asymmetrically distributed returns, application of pure downside risk measures is advantageous. Young (1998) introduces an alternative optimization criterion based on minimum return instead of variance as measure of risk, and proposes a minimax approach. This corresponds to a utility principle with an extreme form of risk aversion on investor's side. Ghaoui et al. (2003) propose a worst-case Value-at-Risk and robustified programming approach based on only partial information about the return distributions, assuming that only bounds on the moments are known. Jarrow and Zhao (2006) apply lower partial moments as risk measure for downside loss aversion and compare the resulting optimal portfolios with the mean–variance based ones. While both methods perform similarly on normally distributed returns, they can lead to significantly different results on returns with asymmetric, heavy-tailed distributions.

1.2. Portfolio optimization based on the Extreme Risk Index (ERI)

In our paper we follow the basic line of developments on the optimization problem that the investment strategy is derived from. Our reformulation of the objective function in this optimization problem is based on extreme value theory, and it is specifically designed for portfolios with heavy-tailed assets. Extreme value theory is an adequate tool to improve the modelling of return tails.

In contrast to the mean–variance optimization, our approach does not rely on existence of second moments for the return distribution. With increasingly heavy tails, variance and covariance estimators can become unreliable, or even the moment themselves may fail to exist. Thus the mean–variance approach tends to face its limitations especially in crisis periods, when financial returns behave in their most extreme way. Several modifications addressing this issue have been discussed; see, e.g., Rachev et al. (2005) for the relevance of this type of heavy-tailed models.

In the present study we apply a novel method based on extreme value theory to a portfolio optimization on real data. This study seems to be the first attempt in extreme-value based portfolio optimization on large scale. Our primary aim is to assess the general potential of extreme-value based methods in portfolio optimization. At this initial stage, we compare a very basic implementation of our extreme-value approach with similarly basic and therefore relatively robust benchmarks. Our benchmarks are given by the minimum-variance portfolio (MV) and the equally weighed portfolio (EW), which invests the $1/N$ fraction of the total capital in each of N assets. According to our results, the extreme-value based method stays behind its benchmarks on assets with light tails, but outperforms each of them (MV and EW) on assets with moderately heavy or very heavy tails. As discussed above, outperforming these simple methods on large scale is non-trivial even with refined estimation techniques. The advantage of the extreme-value based method is particularly strong in the case of heaviest tails, which the method is designed for.

More specifically, the mathematical basis of our approach is laid out in Mainik and Rüschendorf (2010). Our portfolio is obtained by minimizing the Extreme Risk Index (ERI), which quantifies the impact of heavy, dependent tails of asset returns on the tail of the portfolio return. We apply this strategy and the chosen benchmarks to the daily return data of the S&P 500 stocks in the period from November 2007 to September 2011. The computation of portfolio weights utilizes the data from the six years prior to each trading day. To assess the impact of delays in portfolio rebalancing, we implement rebalancing not only on daily, but also on weekly basis. For the sake of stability, the portfolio estimates for both daily and weekly rebalancing are based on daily data. In addition to the portfolio value we also track some other characteristics related to portfolio structure, degree of diversification, and transaction costs.

In the first round of our backtesting experiments we apply ERI optimization to all S&P 500 stocks with full history in our data set (444 out of 500). In this basic setting the ERI based algorithm slightly outperforms the MV and EW portfolios with respect to