Journal of Empirical Finance 25 (2014) 15–35

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Modelling changes in the unconditional variance of long stock return series

Cristina Amado [a,b,*], Timo Teräsvirta [b]

[a] University of Minho and NIPE, Campus de Gualtar, 4710-057 Braga, Portugal
[b] CREATES and Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark

ARTICLE INFO

Article history:
Received 6 March 2012
Received in revised form 22 September 2013
Accepted 25 September 2013
Available online 2 October 2013

JEL classification:
C12
C22
C51
C52
C53

Keywords:
Model specification
Conditional heteroskedasticity
Lagrange multiplier test
Time-varying unconditional variance
Long financial time series
Volatility persistence

ABSTRACT

In this paper we develop a testing and modelling procedure for describing the long-term volatility movements over very long daily return series. For this purpose we assume that volatility is multiplicatively decomposed into a conditional and an unconditional component as in Amado and Teräsvirta (2012, 2013). The latter component is modelled such that the unconditional time-varying component evolves slowly over time. Statistical inference is used for specifying the parameterization of the time-varying component by applying a sequence of Lagrange multiplier tests. The model building procedure is illustrated with an application to 22,986 daily returns of the Dow Jones Industrial Average stock index covering a period of more than ninety years. The main conclusions are as follows. First, the LM tests strongly reject the assumption of constancy of the unconditional variance. Second, the results show that the apparent long memory property in volatility may be interpreted as changes in the unconditional variance of the long series. Finally, based on a formal statistical test we find evidence of the superiority of volatility forecasting accuracy of the new model over the GJR-GARCH model at all horizons for eight subsets of the long return series.

1. Introduction

The observation that deterministic shifts in long return series can generate long memory behaviour has received much attention in recent years. Most of the work in this topic is related with the study of the behaviour of standard statistical tools and model misspecification under nonstationarity. Early studies include Diebold (1986) and Lamoureux and Lastrapes (1990) who suggested that occasional level shifts in the intercept of the first-order GARCH model can bias the estimation towards an integrated GARCH model. More recently, Mikosch and Stărică (2004) argued that the so-called 'integrated GARCH effect' is caused by the nonstationary behaviour of very long return series. They showed how the long-range dependence in volatility and the IGARCH effect may be explained by neglected deterministic changes in the unconditional variance of the stochastic process. Moreover, Granger and Hyung (2004) claimed that occasional breaks in a long time series of absolute stock returns can also explain the observed slow decay of the autocorrelation functions of absolute returns in long return series.

In a standard first-order GARCH model of Bollerslev (1986) the decay rate of the autocorrelation function of squared observations is exponential, which is often considered too rapid. This has motivated the development of more flexible models to describe the observed dependence structure in financial market volatility. One of these models is the Fractionally Integrated

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[*] Corresponding author at: University of Minho and NIPE, Campus de Gualtar, 4710-057 Braga, Portugal. Tel.: +351 253 601 383; fax: +351 253 676 375.
E-mail addresses: camado@eeg.uminho.pt (C. Amado), tterasvirta@econ.au.dk (T. Teräsvirta).

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http://dx.doi.org/10.1016/j.jempfin.2013.09.003

GARCH model of Baillie et al. (1996) which belongs to the class of long memory models. In these processes, the decay rate of the autocorrelations of squared returns is hyperbolic, which often appears more suitable for financial series than the exponential rate of the GARCH model. Baillie and Morana (2009) recently proposed a generalization of the FIGARCH model in which the intercept changes deterministically according to the flexible functional form of Gallant (1984).

The question of explicitly modelling nonstationarity in stock market volatility has received somewhat less attention. van Bellegem and von Sachs (2004) and Feng (2004) proposed decomposing the volatility process multiplicatively into a deterministic nonstationary (or the unconditional variance) and a stochastic stationary (or the conditional variance) component. The deterministic component in their model is estimated nonparametrically. Stărică and Granger (2005) introduced a nonstationary approach in which the returns are modelled as nonstationary sequence of independent random variables with time-varying unconditional variance but their model does not allow for volatility clustering. More recently, Engle and Rangel (2008) and Brownlees and Gallo (2010) applied the same multiplicative decomposition as van Bellegem and von Sachs (2004) and Feng (2004) did. The deterministic nonstationary component is in their approach described by an exponential spline, and the stationary component follows a first-order GARCH process.

This paper addresses the issue of modelling deterministic changes in the unconditional variance of long return series. It is assumed that volatility is modelled by multiplicatively decomposing the variance into a conditional and an unconditional component as in Amado and Teräsvirta (2008); for more recent versions, see Amado and Teräsvirta (2012, 2013). The conditional variance follows a GARCH process and describes the short-run dynamics of volatility. The nonstationary component of volatility describes the long-volatility dynamics, and it is represented by a linear combination of logistic transition functions. Statistical inference is used for specifying the parametric structure of the time-varying component by applying a sequence of Lagrange multiplier tests.

Our modelling strategy is applied to describe the long-run properties of the long daily Dow Jones Industrial Average (DJIA) return series from 1920 to 2011. One may expect that the longer the observation period, the more likely the occurrence of structural changes or shifts in the second unconditional moment of returns. This requires modifications to the modelling strategy of Amado and Teräsvirta (2008, 2012) that works well when one can at most expect, say, three or four shifts in the series. These modifications, based on dividing the long series into subseries, applying the strategy to them and combining the results into a single model, form the methodological contribution of this paper.

The results of this modified specification search strongly support the time-variation of the unconditional variance in the period under study. The estimation results indicate that the strongest deterministic changes in the unconditional variance are associated with the largest economic recessions. From the modelling point of view, our findings suggest that the observed long memory property in volatility may well be due to deterministic changes in the unconditional variance of the return series. Moreover, the out-of-sample forecasting accuracy of the proposed model is also studied over several forecasting horizons. Modelling the long-term volatility movements over a long return series generates more accurate volatility forecasts than the GJR-GARCH model for all horizons. However, the predictive ability of the new model is strikingly improved across all horizons by discarding the old observations and merely using the most recent observations (about one-tenth of the total) to build the model for out-of-sample forecasting.

The paper is organized as follows. The TV-GARCH model and the modelling strategy are presented in Section 2. The details regarding the estimation of the model are discussed in Section 3. Section 4 contains the application and Section 5 concludes.

2. A model for the long-term volatility component

2.1. The time-varying GARCH framework

In this paper the tool for modelling an asset return series over a long period is a GARCH-type model in which the unconditional variance is assumed to evolve smoothly over time. We begin by focusing on the long-run properties of the GJR-GARCH (p, q) model of Glosten et al. (1993). Let F_{t-1} be the information set containing the historical information of the series of interest available at time t-1 and write the asset returns $\{y_t\}$ as

$$y_t = \mathrm{E}(y_t|F_{t-1}) + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \zeta_t h_t^{1/2} \tag{2}$$

where $\{\zeta_t\}$ ~ nid (0,1). Under this assumption the conditional distribution of the innovation sequence $\{\varepsilon_t\}$ is $\varepsilon_t|F_{t-1} \sim N(0,h_t)$. For simplicity, the conditional mean of the asset returns is set equal to zero, i.e., $\mathrm{E}(y_t|F_{t-1}) = 0$. The component h_t describes the dynamics of the conditional variance of the asset returns. To allow positive and negative shocks to have an asymmetric effect on the stock market volatility we choose the GJR-GARCH(p, q) model for h_t. It has the form

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \kappa_i \varepsilon_{t-i}^2 I(\varepsilon_{t-i}<0) + \sum_{j=1}^{p} \beta_j h_{t-j}. \tag{3}$$