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SCIENCE DIRECT®
Journal of Empirical Finance 13 (2006) 113–128
Journal of EMPIRICAL FINANCE
www.elsevier.com/locate/jempfin

A re-examination of the asymmetric power ARCH model ☆

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Accepted 3 May 2005

Abstract

The purpose of this paper is to provide a comprehensive methodology for the analysis of the Asymmetric Power ARCH model. First, it gives the ARMA representations of a power transformation of the conditional variance and the absolute returns. Second, it derives a certain fractional moment of the absolute observations. Third, it obtains the autocorrelation function of the power-transformed absolute returns. Finally, the practical implications of the results are illustrated empirically using daily data on five East Asia stock indices.

JEL classification: C13; C22; C52
Keywords: Asymmetric power ARCH; Power transformation; Autocorrelations; Stock returns

1. Introduction

A common finding in much of the empirical finance literature is that although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated (see, for example, Taylor, 1986; Ding et al., 1993; Granger and Ding, 1995; Ding and Granger, 1996). In particular, Ding et al. (1993) investigate the autocorrelation structure of $|r_t|^d$, where r_t is the daily S&P 500 stock market returns, and d is a positive number. They found that $|r_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which

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0927-5398/$ - see front matter
doi:10.1016/j.jempfin.2005.05.002

they call the Asymmetric Power ARCH (A-PARCH) model. In addition, they show that the A-PARCH model comprises seven other models in the literature. He and Teräsvirta (1999b) illustrate how the A-PARCH model may also be viewed as a standard GARCH model for observations that have been transformed by a sign-preserving power transformation implied by a (modified) A-PARCH parameterization.

The purpose of this paper is to study the autocorrelation structure of the general A-PARCH(p,q) model. The moment structure of the GARCH family of models is a topic that has recently attracted a great deal of attention. Karanasos (1999) and He and Teräsvirta (1999a) derived the autocorrelations of the squared errors for the GARCH model.[1] In addition, He and Teräsvirta (1999b), using the above sign-preserving transformation, obtained the autocorrelation function of the power-transformed absolute errors for the first-order A-PARCH model. Despite this progress, the moment structure of the A-PARCH(p,q) model has not been fully worked out yet.

In this paper we view the A-PARCH model from a different angle, and provide a comprehensive methodology for the analysis of the general A-PARCH(p,q) process. First, we give the ARMA representations of the power transformations of the conditional variance and the absolute returns. Next, we derive an existence condition for a certain fractional moment of the absolute observations. The practical significance of the existence condition for a fractional moment is that when it is satisfied, then all lower-order moments exist as well. In contrast, violation of the above condition implies that no higher-order moments exist.[2] Further, we obtain the autocorrelation function of the power-transformed absolute returns. Our results on the moment structure of the general A-PARCH(p,q) model extend the results in He and Teräsvirta (1999b) on the first-order A-PARCH model, and Karanasos (1999) and He and Teräsvirta (1999a) on the GARCH(p,q) model.

Several previous articles dealing with financial market data–e.g., Dacorogna et al. (1993), Ding et al. (1993) and Muller et al. (1997)–have commented on the behaviour of the autocorrelation function of power-transformed absolute returns, and the desirability of having a model which comes close to replicating certain stylized facts in the data (abstracted from Baillie and Chung, 2001). In this respect, estimates of the autocorrelations of power-transformed observations can be of great importance. By comparing these estimates to those obtained by the data, one can have a clear indication of how well the estimated model fits the data.

Another potential motivation for the derivation of the autocorrelations of the power-transformed absolute returns is that they can be used to estimate the parameters of the A-PARCH model. The approach is to use the minimum distance estimator (MDE), which estimates the parameters by minimizing the Mahalanobis generalized distance of a vector of sample autocorrelations from the corresponding population autocorrelations (see Baillie and Chung, 2001).[3] In a recent paper, Kristensen and Linton (in press) propose a closed-form estimator for

[1] Theoretical results on the moment structure of the EGARCH model have also been derived (see Demos, 2002; He et al., 2002; Karanasos and Kim, 2003). Further, for a discussion of the GARCH-in-mean model, see Karanasos (2001), Arvanitis and Demos (2004) and Karanasos et al. (2004).

[2] Ling and McAleer (2002a) provide the necessary and sufficient condition for the existence of higher order moments of a version of the A-PGARCH(p,q) model. Ling and McAleer (2002b) investigate some structural properties of a family of GARCH(1,1) processes.

[3] One motivation for the MDE approach can be found in Jacquier et al. (1994) who, on examining the autocorrelations of transformations of fitted returns from maximum likelihood estimation (MLE), have noted their discrepancy when compared with the autocorrelations of actual returns.