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Optimal conditional hedge ratio: A simple shrinkage estimation approach

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ABSTRACT

A number of recent studies adopt bivariate generalized autoregressive conditional heteroskedasticity (BGARCH) models to estimate the optimal conditional hedge ratio. Since the optimal hedge ratio can be expressed by the ratio of variance of futures returns to the covariance of spot and futures, the BGARCH model is quite useful to estimate the conditional hedge ratio. However, it is well known that high variability of an estimated conditional hedge ratio results in lower hedge effectiveness. In this study, we consider a simple shrinkage method to deal with this inverse relationship between volatility of the conditional hedge ratio and hedging effectiveness. Our main idea is that the shrinkage version of the optimal hedge ratio can be obtained from a convex combination of unconditional sample covariance matrix and conditional covariance matrices of a conventional BGARCH model. Our empirical results show the usefulness of our proposed model.

1. Introduction

In the empirical finance literature, a number of models exist that estimate the optimal time-varying hedge ratio using futures prices; for example, the bivariate autoregressive conditional heteroskedasticity (ARCH) model (Cecchetti et al., 1988), the bivariate generalized autoregressive conditional heteroskedasticity (BGARCH) model (Baillie et al., 2007, Baillie and Myers, 1991, Bera et al., 1997, Lien et al., 2002, Myers and Thompson, 1989, Park and Jei, 2010), the random coefficient model (Bera et al., 1997), and the vector error correction model (VECM) (Brooks et al., 2002, Lien and Yang, 2006). Time-varying hedge ratios are usually referred to as conditional hedge ratios in the literature since the estimated hedge ratios are conditioned on the data set available for the previous time period. Recently, more sophisticated and flexible models are used to improve hedging effectiveness; for example, a random coefficient autoregressive Markov regime switching model (Lee et al., 2006), a copula-based generalized autoregressive conditional heteroskedasticity (GARCH) model (Hsu et al., 2008), a wavelet-based model (Conlon and Cotter, 2012), a Markov regime-switching autoregressive moving-average (ARMA) model (Chen and Tsay, 2011), a higher-order moment model (Brooks et al., 2012), a stochastic volatility model (Liu et al., 2014), and a functional coefficient model (Fan et al., 2015). However, the out-of-sample performances of most of these models are very similar to or dominated by a simple

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constant (unconditional) hedging strategy obtained by the ordinary least squares (OLS) estimate for the slope parameter in the linear regression of spot returns on futures returns.

The optimal conditional hedge ratio is basically represented by the conditional covariance between spot and futures returns divided by the conditional variance of futures returns. Thus, different conditional covariance and variance estimates naturally yield different optimal conditional hedge ratios, and many studies provide different estimation strategies to obtain a well-performing conditional hedge ratio. In this study, we propose a model based on some past empirical findings on hedging effectiveness. Park and Jei (2010) forecast future hedge ratios by using a bootstrap method and comparing the bootstrap variance of the estimated hedge ratios with out-of-sample hedging performance. They find a negative relationship between volatility of the hedge ratio and hedging effectiveness. Moreover, when the estimated hedge ratios are highly persistent, the out-of-sample performances of these hedge ratios are better than those of the unconditional hedge ratios. Lien (2010a) also shows the negative relationship between hedge ratio volatility and hedging performance theoretically. In order to obtain an optimal conditional hedge ratio with a small degree of volatility but that preserves the persistency of a conditional hedge ratio estimated from a simple and widely used model, we propose a shrinkage-type model.

Our proposed conditional hedge ratio, referred to as the conditional shrinkage hedge ratio, is represented by a convex combination of the conditional hedge ratio from a BGARCH model and an unconditional hedge ratio. Thus, we interpret our proposed conditional hedge ratio as a hedge ratio that shrinks the unconditional hedge ratio into the conditional hedge ratio. This specification automatically yields a hedge ratio with variance smaller than that of the conditional hedge ratio. The shrinkage characteristics of the proposed hedge ratio come from shrinkage interpretation of the covariance matrix. Shrinkage intensity (a weight) is selected such that it maximizes the variance reduction measure, which is the most frequently used hedging performance measure in the literature. The conditional shrinkage hedge ratio has at least three advantages over the existing conditional hedge ratios: (i) since the weight of the conditional hedge ratio in the convex combination framework is time-varying, the resulting conditional shrinkage hedge ratio is not as volatile as the conditional hedge ratio, and, moreover, it is persistent; (ii) the conditional shrinkage hedge ratio maximizes the variance reduction measure of all models belonging to the convex combination of conditional and unconditional covariance matrices; and (iii) the computational cost is much smaller than that of numerous existing models. To estimate the conditional shrinkage hedge ratio, we need to estimate the BGARCH model and maximize the variance reduction measure. The former can be done in a few seconds with a simple BGARCH model. The latter can be easily done by a grid search since the shrinkage intensity is given by a scalar.

In the empirical part of this study, by using the futures prices of financial products, we show that the estimated conditional shrinkage hedge ratios are much less volatile than those estimated using the BGARCH model. For evaluation of the out-of-sample hedging performance, we split the sample into three sub-periods and show that the proposed conditional shrinkage hedge ratio performs relatively well. We also consider a one-step-ahead forecast of hedge ratios using the rolling window scheme to evaluate out-of-sample hedging performance and find that the conditional shrinkage hedge ratios dominate other conventional hedge ratios.

The rest of the paper is organized as follows. Section 2 introduces the shrinkage model we propose in this study. The data used and their descriptive statistics are described in Section 3. Section 4 reports the estimation results and compares different hedging performances. Finally, Section 5 concludes the paper.

2. The econometric model

A number of econometric models estimate the conditional optimal hedge ratio. The optimal conditional hedge ratio minimizes the conditional risk of hedged portfolio returns and can be expressed as

$$\beta_t = \frac{\mathrm{Cov}\left(R_t^s, R_t^f | \mathcal{F}_{t-1}\right)}{\mathrm{Var}\left(R_t^f | \mathcal{F}_{t-1}\right)}, \tag{2.1}$$

where R_t^s and R_t^f are the spot and futures returns and $\mathcal{F}_{t-1}$ denotes the σ-algebra generated by using all the available information up to time $t-1$. One of the most frequently used methods is a BGARCH model, with which the conditional covariance, $\mathrm{Cov}\left(R_t^s, R_t^f | \mathcal{F}_{t-1}\right)$, and variances, $\mathrm{Var}\left(R_t^f | \mathcal{F}_{t-1}\right)$, of the spot and futures returns can be easily estimated. In general, the BGARCH regression model can be written as follows:

$$R_t = m_t(\zeta) + \epsilon_t, \tag{2.2}$$

$$\epsilon_t | \mathcal{F}_{t-1} \sim F(0, H_t), \tag{2.3}$$

where $R_t = \left(R_t^s, R_t^f\right)'$, $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$, and $m_t(\zeta) = (m_{1t}(\zeta_1), m_{2t}(\zeta_2))'$ are vector-valued conditional mean functions, $\zeta = (\zeta_1, \zeta_2)'$ is a $p \times 2$ conditional mean parameter, F denotes a bivariate distribution, and H_t is a time-varying 2×2 positive definite conditional covariance matrix. The above models (2.2) and (2.3) can be estimated by a maximum likelihood estimation method if