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Testing of a market fraction model and power-law behaviour in the DAX 30

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ABSTRACT

This paper tests a simple market fraction asset pricing model with heterogeneous agents. By selecting a set of structural parameters of the model through a systematic procedure, we show that the autocorrelations (of returns, absolute returns and squared returns) of the market fraction model share the same pattern as those of the DAX 30. By conducting econometric analysis via Monte Carlo simulations, we characterize these power-law behaviours and find that estimates of the power-law decay indices, the (FI)GARCH parameters, and the tail index of the selected market fraction model closely match those of the DAX 30. The results strongly support the explanatory power of the heterogeneous agent models.

1. Introduction

Traditional economic and finance theory is based on the assumptions of investor homogeneity and rational expectations. Since agents are rationally impounding all relevant information into their trading decisions, the movement of prices is assumed to be perfectly random and hence exhibit random walk behaviour. This view is the theoretical underpinning of the efficient market hypothesis and asset pricing theories generally either implicitly or explicitly, including the optimal portfolio rules developed by Markowitz (1952) and Merton (1971), the static and intertemporal capital asset pricing model of Sharpe (1964), Lintner (1965), Mossin (1966) and Merton (1973a) and models for the pricing of contingent claims beginning with the work of Black and Scholes (1973) and Merton (1973b). The impressive statistical evidence in favour of market efficiency, documented by Fama (1976), has been taken as support for the random walk model and for a long time financial economists were contented with this view as the explanation of the time series behaviour of observed asset prices.

Empirical investigations of (high-frequency) financial time series in both equity and foreign exchange markets show some common features that are not in line with these assumptions, so-called stylized facts: include excess volatility (relative to the dividends and underlying cash flows), excess skewness, fat tails (the tails of distribution have a higher density than that predicted by the normal distribution, which conventionally was indicated by excess kurtosis), volatility clustering (high/low fluctuations are followed by high/low fluctuations), and long-range dependence in volatility (often characterized by slow decay of autocorrelations of squared or

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absolute returns. More precisely, we see insignificant autocorrelations (ACs) of raw returns and hyperbolic decline of ACs of the absolute and squared returns, see Ding et al., 1993), and various power-law behaviour. We refer to Pagan (1996) for a comprehensive discussion of stylized facts characterizing financial time series and Lux (2008) for a recent survey on empirical evidence of various power laws. These facts are not entirely contradictory to the traditional economic and finance theory with representative agent and rational expectations, but the theory does not provide persuasive explanation on a large subset of these facts. Our paper contributes to the explanation of the stylized facts from a heterogeneous agent models (HAMs) perspective.

Our paper considers a simple market fraction (MF) model developed by He and Li (2007, 2008). The MF model is a simple stochastic asset pricing model, involving two types of traders (fundamentalists and trend followers) under a market maker scenario. He and Li (2008) describe various aspects of financial market behaviour and establish the connection between the stochastic model and its underlying deterministic system. Through a statistical analysis, He and Li (2008) show that convergence of market price to fundamental value, long- and short-run profitability of the two trading strategies, survivability of trend followers and various under- and over-reaction autocorrelation patterns of the stochastic model can be explained by the dynamics, including the stability and bifurcations, of the underlying deterministic system. Based on these results, He and Li (2007) study the generating mechanism of the MF model to produce the volatility clustering and the long-range dependence in volatility. The results show that heterogeneity, risk-adjusted trend chasing, and the interplay of a stable deterministic equilibrium and stochastic noisy processes can be the source of power-law distributed fluctuations. The power-law behaviour is further verified by econometric estimates via a Monte Carlo simulation. The analysis of generating mechanism and power-law decay estimation based on simulations in He and Li (2007) provide a promising perspective for testing of the MF model to actual data in this paper.

In this paper, we test the MF model using the daily DAX 30 index and pay particular attention on the power-law behaviour in volatility. By selecting a set of structural parameters of the model through a systematic procedure, we show that the autocorrelations of returns, absolute returns and squared returns of the MF model share the same patterns for the DAX 30. By conducting econometric analysis via Monte Carlo simulations, we then characterize these power-law behaviours and find that estimates of the power-law decay indices, the (FI)GARCH parameters, and the tail index of the selected market fraction model match closely to the corresponding estimates for the DAX 30. Interpretation of the selected parameters is consistent with the power-law behaviour generating mechanism in He and Li (2007). The results provide very positive evidence on the explanatory power of heterogeneous agent models.

We should emphasize that econometric analysis, especially estimation of heterogeneous agent models is still a challenging task, see for instance, a recent comprehensive review by Chen et al. (2012). Generally, the difficulties of estimation come from the complexity of the HAMs, together with (typically) many parameters, which makes verification of identification rather difficult, and thus proving consistency of estimation troublesome. For recent attempts to estimate HAMs, the identification problem is typically circumvented by focussing on a relatively simple HAMs, or by estimating a few key parameters only. Boswijk et al. (2007) derive a reduced form equation from a simplified Brock and Hommes (1997, 1998) type model and estimate it by using nonlinear least square method. Alfarano et al. (2005) estimate a simplified herding model by maximum likelihood method. Amilon (2008) estimates two specifications of the extended Brock and Hommes switching models described in De Grauwe and Grimaldi (2003, 2006) by using the efficient method of moments and maximum likelihood method. He concludes that the simple prototype models he estimated seem to have potential to explain empirical facts although the fit is generally not quite satisfactory. But he also reports local minima, possibly not the global minimum, when calculating the estimators. Franke (2009) applies the method of simulated moments to a model developed by Manzan and Westerhoff (2005). He reports that one of the parameters is not identified. He also discusses the problem of many local minima. Nevertheless, the other (identified) parameters could be estimated in a meaningful way, despite the simplicity of the model. Thus, although good progress seems to be made in estimating HAMs, even in case consistent estimation would be possible, the likely heavily nonlinear relationship between observables and unknown parameters to be estimated might seriously complicate estimation, see, for example, Chen et al. (2012), and experienced by, for example, Amilon (2008) and Franke (2009). Therefore, in this paper, following Li et al. (2010) we select structural parameters of interests in the MF model class that minimizes a distance between data based and HAMs based parameters.

Our approach seems relevant to general HAMs, particularly when dealing with more complicated models. Moreover, quite possibly a HAM might be misspecified, so that likelihood and/or moments based methods might produce poor results. For instance, this might be the reason that in Franke (2009) the method of simulated moments, using the optimal weighting matrix, produces less favourable results compared to estimating the parameters based on an intuitive way of linking theoretical and empirical moments. Our procedure is more robust to model misspecification, sharing the same spirit as real business cycles literature, such as Kydland and Prescott (1982), and equity premium puzzle literature, Mehra and Prescott (1985), Diebold et al. (1998) and Schorfheide (2000) in the macroeconomics literature, Gilli and Winker (2003), and Winker and Gilli (2003) in agent-based financial market literature.[1]

The remainder of the paper is organized as follows. Section 2 provides a selective literature review on the development of HAMs. Section 3 summaries the MF model. In Section 4 we first systematically select a set of structural parameters of the MF model to characterize the power-law behaviour in volatility of the DAX 30 stock market daily closing price index. We then estimate the power-law decay parameters of the autocorrelation of returns, the squared returns and the absolute returns, (FI)GARCH (1, 1) parameters, and the power-law decay rates of the tail distribution for both the DAX 30 index and MF model-generated data. In Section 5, we present an explanation of the selected structural parameters of the MF model. Section 6 concludes.

[1] For the discussions on comparison of this methodology with the usual estimation methodology, see, e.g., Canova (1994), Hansen and Heckman (1996), Kydland and Prescott (1991, 1996), Geweke (2006), and Dridi et al. (2007). This methodology is also closely linked to Indirect Inference (Dridi and Renault, 2000; Dridi et al., 2007; Gourieroux et al., 1993).