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It's all about volatility of volatility: Evidence from a two-factor stochastic volatility model☆

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ABSTRACT

The persistent nature of equity volatility is investigated by means of a multi-factor stochastic volatility model with time varying parameters. The parameters are estimated by means of a sequential matching procedure which adopts as an auxiliary model a time-varying generalization of the HAR model for the realized volatility series. It emerges that during the recent financial crisis the relative weight of the daily component dominates over the monthly term. The estimates of the two factor stochastic volatility model suggest that the change in the dynamic structure of the realized volatility during the financial crisis is due to the increase in the volatility of the persistent volatility term. A set of Monte Carlo simulations highlights the robustness of the methodology adopted in tracking the dynamics of the parameters.

1. Introduction

The aim of this paper is to evaluate whether the observed changes in the dynamic behavior of the realized volatility (RV) series, in correspondence to the financial crises, are linked to changes in the structural parameters governing the stochastic volatility (SV) dynamics in continuous time. In other words, the observed changes in the dynamic pattern of the RV series during the financial crises may be seen as the outcome of structural breaks in the parameters governing the dynamics of the continuous-time SV process. The volatility dynamics are assumed to be driven by a two factor SV model (TFSV) that successfully accounts for the long range dependence of the volatility process, as noted by Gallant et al. (1999) and Meddahi (2002, 2003). Given the difficulty of a direct estimation of breaks in the TFSV parameters, we adapt the indirect inference procedure suggested by Corsi and Renó (2012) to the case of a recursive updating of the SV parameters. The proposed method exploits a flexible specification for the auxiliary model, built on an ex-post measure of the integrated variance, IV. The auxiliary model is a time varying extension of the well-known HAR model of Corsi (2009), and it represents a tool to evaluate as to what extent the parameters governing the dynamic structure of the RV process vary over time. The time-varying HAR (TV-HAR) is interesting per se as it constitutes a tool to evaluate the evolution of the relative

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weight of each volatility component to the overall volatility persistence. Following Raftery et al. (2010) and Koop and Korobilis (2012), we use a fast *on-line* method to extract the TV-HAR parameters, allowing for a rapid update of the estimates as each new piece of information arrives. The advantage of the proposed estimation method is that it does not require identifying the number of change points and avoids the use of computationally intensive algorithms, such as MCMC.

The empirical analysis is carried out on the volatility series of 15 assets traded on the NYSE, that are representative of the main sectors of the US economy. The estimates of the TFSV model, based on a sequential matching of the TV-HAR parameters, clearly indicate the instability of the TFSV parameters. The main finding is that the parameters governing the speed of mean reversion and the volatility of volatility of the persistent factor display a significant dynamic behavior. Specifically, the speed of mean reversion drops during the financial crisis, while the volatility of volatility increases, especially for the assets belonging to the bank and financial sectors. As a consequence, the change in persistence in the volatility series can be attributed to the increase of the relative weight of the persistent volatility component. The relative increase of the persistent volatility factor generates trajectories that deviate for longer periods from the unconditional mean, hence producing the impression of level shifts in the observed RV series. Moreover, the higher volatility of the persistent volatility factor increases the degree of dispersion of the volatility around its long-run value, and thus the volatility of RV (see Corsi et al., 2008). Interestingly, the growth of the volatility of the persistent factor is reflected in an increase of the relative weight of the daily volatility component in the auxiliary TV-HAR model. In particular, the daily term becomes the main factor during the financial crisis. On the other hand, the monthly component has a larger role during the low volatility period which characterizes the years 2004–2007.

Summarizing, the contributions of this paper are at least threefold. First, the TV-HAR model is proposed as a flexible tool to describe and predict the dynamics of the RV series, and a fast *on-line* estimation technique for the model parameters is adopted. The in-sample estimates of the TV-HAR clearly display a large degree of instability in the autoregressive terms, especially during the financial crisis period, as a consequence of the change in the persistence of the RV. Although this possibility is not explored in the present paper, the TV-HAR model coupled with the fast Kalman filter updating and dynamic model averaging could be exploited to obtain precise real-time out-of sample forecasts and to the optimal dynamic selection of the significant autoregressive terms. Secondly, the model selection procedure, based on the predictive likelihood, excludes that breaks in the long-run mean during the financial crisis are responsible for the increase in the observed persistence of the volatility series. This is an interesting result given the recent debate on the possible confusion between long-memory and level-shifts in the RV series. The results indicate that the increase in the persistence of RV during the financial crisis is more likely to be caused by a variation in the autoregressive roots of RV than by a break in its long run mean. Finally, the sequential estimates of the TFSV parameters indicate instability in the model parameters. The updated estimates of the TFSV model, conveniently corrected for risk premiums, could be used for option price purposes and to analyze the evolution through time of the volatility smile.

The paper is organized as follows. First, Section 2 introduces the auxiliary TV-HAR model. Section 3 sets the notation of the TFSV model and proposes a dynamic matching method for the TFSV model using the TV-HAR as an auxiliary model. Section 4 presents the results of the empirical analysis based on the 15 stocks traded on NYSE. Section 5 provides Monte Carlo simulations to evaluate the robustness of the empirical results presented in Section 4 and the possible presence of leverage effects and jumps. Section 6 concludes.

2. Auxiliary model: the TV-HAR

A strong empirical evidence, dating back to the seminal papers of Engle (1982) and Bollerslev (1986), supports the idea that the volatility of financial returns is time varying, stationary and long-range dependent. This evidence is confirmed by the statistical analysis of the ex-post volatility measures, such as RV, that are precise estimates of latent integrated variance and are obtained from intradaily returns, see Andersen and Bollerslev (1998), Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) and among many others. In the last decade, particular effort has been spent in developing discrete time series models for ex-post volatility measures, that are able to capture the persistence of the *observed* volatility series.[2] Reduced form time series models for RV have been extensively studied during the last decade. For instance, Andersen et al. (2003), Giot and Laurent (2004), Lieberman and Phillips (2008) and Martens et al. (2009) report evidence of long memory and model RV as a fractionally integrated process. As noted Ghysels et al. (2006) and Forsberg and Ghysels (2007) and mixed data sampling approaches are also empirically successful in accounting for the observed strong serial dependence. In particular, Corsi (2009) approximates the long range dependence by means of an autoregressive process with many lags, called heterogeneous-autoregressive model (HAR). The main feature of the HAR model is its interpretation as a volatility cascade, where each volatility component is generated by the actions of different types of market participants with different investment horizons. HAR type parameterizations are also suggested by Corsi et al. (2008), Andersen et al. (2007) and Andersen et al. (2011).

In its simplest version, the HAR model of Corsi (2009) is defined as

$$X_t = \alpha + \phi^d X_{t-1} + \phi^w X_{t-1}^w + \phi^m X_{t-1}^m + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), \tag{1}$$

where $X_t = log(RV_t), X_t^w = \frac{1}{5}\sum_{j=0}^{4} X_{t-j}, X_t^m = \frac{1}{22}\sum_{j=0}^{21} X_{t-j}$. It is clear that the HAR model is a AR(22) with linear restrictions on the autoregressive parameters. In particular, there are three free parameters with an autoregressive equation with 22 lags. Corsi et al.

[2] Recent papers by McAleer and Medeiros (2011) and Asai et al. (2012) present detailed surveys of alternative models for RV.