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On the distribution and estimation of trading costs

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ABSTRACT

This paper investigates the uncertainty about the trading costs associated with a given portfolio strategy. I derive accurate approximations of the ex ante probability distributions of proportional trading costs and portfolio turnover under the conventional assumption of normal asset returns. Based on these approximations, I express the expected trading costs as a function of asset and portfolio characteristics. All else equal, the expected trading costs increase with: i) the deviations of the expected asset returns from the expected portfolio return, ii) the assets' volatility and iii) the portfolio volatility. At the same time, they decrease with the covariance between the assets and the portfolio. Furthermore, I propose novel estimators of the expected turnover and trading costs and show that they offer small bias and low variance, even when the sample size is small. Finally, I incorporate my results into a portfolio selection framework to produce portfolios with low levels of risk and trading costs. Several experiments with real and simulated data confirm the practical value of the results.

1. Introduction

Trading costs, such as bid–ask spreads, brokerage fees, price impact and taxes, play a central role in financial markets. For example, in his 2008 American Finance Association presidential address, Kenneth French reported that U.S. investors paid $32.1 billion in trading costs in 2006 (French, 2008). Because trading costs have a diminishing effect on investment returns, it is important to incorporate them in the portfolio choice process. However, these costs are not known before the time of the trade, as they are a function of market prices that are also unknown in advance. This paper sheds some light on this problem by studying the ex ante probability distribution of proportional trading costs for a given portfolio strategy.[1]

While several studies explore the probability distribution of the portfolio weights, mean return, variance and Sharpe ratio (e.g., Kan and Smith, 2008; Kan and Zhou, 2007; Kourtis et al., 2012; Ledoit and Wolf, 2008; Okhrin and Schmid, 2006), the ex ante distribution of trading costs remains unknown in the literature despite their importance in optimal portfolio choice.[2] Understanding the distributional properties of trading costs can lead to answers to three central questions: i) What is the connection between trading costs and the returns on the assets that form the portfolio? ii) How can one estimate trading costs efficiently? iii) How can one improve the management of trading costs?

I study the proportional trading costs paid by an investor that rebalances to a desired target portfolio at the end of each period. The target is known to the investor ex ante. While this framework can accommodate target portfolios that change from period to period, to

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[1] Although the literature has also considered fixed or quadratic trading costs, proportional costs are the most common model (e.g., see Balduzzi and Lynch, 1999; Davis and Norman, 1990; Magill and Constantinides, 1976, among several others). This model directly corresponds to the bid–ask spread and brokerage fees.

[2] For example, Kourtis (2013) shows that most theory-based portfolio strategies underperform naive diversification when trading costs are taken into account.

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ease exposition I focus on the special case that the portfolio weights are fixed over time. Despite the well-known benefits of dynamic portfolio choice, fixed-weight portfolio strategies are still popular among investment management practitioners (e.g., see Amenc et al., 2011; Qian, 2014; Sharpe, 2010). For example, a large portion of professional investors tend to use mean-variance optimization (Amenc et al., 2011) or naive methods, such as equal weights (Benartzi and Thaler, 2001).[3]

My analysis is based on an accurate approximation of the distribution of portfolio turnover. Portfolio turnover measures the amount of trading that the implementation of a portfolio strategy generates in a given period. As such, high levels of turnover are associated with large trading costs. Under the conventional assumption of normal returns, I show that portfolio turnover is approximately distributed as a sum of *folded normal* variables, where folded normal is the distribution of the absolute value of a normal variable.[4] On this basis, I similarly show that proportional trading costs can also be approximated as a sum of folded normal distributions. A numerical experiment confirms that the approximations are highly accurate.

These results allow me to express the expected portfolio turnover and trading costs as intuitive functions of the moments of asset and portfolio returns. In particular, I analytically show that, all else equal, the average trading costs and the average turnover increase with: i) the deviation of the expected asset returns from the expected portfolio return, ii) the assets' volatility and iii) the portfolio volatility. I also find that as the correlation between an asset and the portfolio increases, the expected trading costs decrease. These findings provide clear guidelines to the portfolio manager regarding the asset selection process under trading costs. To the best of my knowledge, this is the first study that connects the asset characteristics with the average trading costs using analytical expressions.

Although a large strand in the literature provides estimates of the actual trading costs paid by investors (e.g., Bhardwaj and Brooks, 1992; French, 2008; Lesmond et al., 1999; Stoll and Whaley, 1990), there is a lack of research into estimating the trading costs in advance. In this context, I use the analytical expressions of the expected trading costs and turnover to develop novel estimators. It is common in practice for the portfolio manager to have views on the expected returns, risk and covariance of the assets or to estimate these using historical data. I show how such estimates can be directly applied to estimate the portfolio turnover and trading costs. The results from several datasets of simulated and real returns indicate that the new estimators offer a low bias as well as small variance, even when the sample size is small. As such, they can be directly employed in practice to accurately forecast the trading costs a given portfolio strategy is expected to produce.

In most of this study, I assume symmetric costs, i.e., the proportional cost of selling is the same as the cost of buying. In practice, these costs may differ (e.g., if there are capital gains taxes). I account for this scenario by separately computing the expected selling costs and the expected buying costs and studying their relation with the asset and portfolio characteristics. A useful implication of these results is an intuitive closed-form expression of the expected capital gains taxes.

The results in this study also have straightforward application to two popular portfolio rebalancing strategies, i.e., periodic rebalancing (e.g., Donohue and Yip, 2003) and threshold rebalancing (e.g., Masters, 2003).[5] The first approach advocates rebalancing the portfolio to the target in fixed time intervals, e.g., every month, quarter or year. In this context, the estimators in this paper can be applied to compute the expected trading costs from periodic rebalancing in a given period. The second approach rebalances the portfolio only when one of the weights deviates from the target weight more than a given threshold. Although the expected trading costs have a more complicated form in this case, I provide a way to estimate the ex ante probability that an asset will trigger rebalancing.

Finally, I incorporate the formulas of the expected trading costs derived in this paper into a portfolio selection model. I consider a multiperiod investor who selects her portfolio to minimize a tradeoff between the error of tracking the target portfolio and the expected transaction costs. A numerical experiment shows that the resulting portfolio offers low turnover and trading costs and, in several cases, improves the net portfolio return. At the same time, it produces low levels of tracking error.

Overall, the contribution of this study is six-fold. First, it provides accurate closed-form approximations of the distributions of the portfolio turnover and trading costs. Second, it presents the effects of the asset and portfolio characteristics on the trading costs using analytical expressions. Third, it derives new estimators of the expected turnover and trading costs that offer low bias and variance. Fourth, it accounts for asymmetric trading costs and capital gains taxes. Fifth, it provides an estimation procedure for the probability that an asset will trigger rebalancing when a threshold exists. Sixth, it develops a portfolio selection framework that yields portfolios that simultaneously control the levels of tracking error and expected trading costs.

The remainder of the paper is organized as follows. Section 2 presents the investment framework, the datasets and the benchmark strategy I employ in most examples in this work. The probability distributions of the portfolio turnover and the trading costs are derived in Section 3. Section 4 connects the expected trading costs with the asset and portfolio characteristics. Section 5 improves the intuition of the results by studying the special case of two assets. Section 6 develops a new type of estimators for the expected turnover and trading costs and assesses its performance in several datasets of simulated and real returns. A discussion of how the analysis is extended in the case of asymmetric trading costs is included in Section 7. The application of my results to two common rebalancing strategies is discussed in Section 8. Section 9 incorporates the formula of the expected trading costs of this work into a portfolio selection framework, while Section 10 concludes this paper.

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[3] Tu and Zhou (2011) attribute the rare use of dynamic models in practice to the fact that intertemporal hedging demands are generally small and to the difficulty in solving the dynamic portfolio choice problem. Amenc et al. (2011) also mention, as a potential reason, the lack of a consensus about the most effective dynamic model.

[4] The folded normal distribution was first studied by Leone et al. (1961). Other relevant contributions include Elandt (1961), Johnson (1962), Psarakis and Panaretos (2001) and Chakraborty and Chatterjee (2013).

[5] Several studies provide superior rebalancing strategies, however they are generally difficult to apply for more than two assets (e.g., Davis and Norman, 1990; Dumas and Luciano, 1991). Exceptions include the models of Leland (1999) where an approximate solution is feasible numerically; that of Liu (2004) which assumes uncorrelated returns; and Dybvig's (2005) model, which assumes a one-period setting. Woodside-Oriakhi et al. (2013) provide a recent review of this literature.