Journal of Empirical Finance 28 (2014) 352–361

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Intraday periodicity adjustments of transaction duration and their effects on high-frequency volatility estimation

Yiu-Kuen Tse *, Yingjie Dong

School of Economics, Singapore Management University, Singapore

A R T I C L E I N F O

Article history:
Received 10 April 2013
Received in revised form 18 April 2014
Accepted 22 April 2014
Available online 28 April 2014

JEL classification:
C410
G120

Keywords:
Autoregressive conditional duration model
Intraday volatility
Time transformation
Transaction data

A B S T R A C T

We study two methods of adjusting for intraday periodicity of high-frequency financial data: the well-known Duration Adjustment (DA) method and the recently proposed Time Transformation (TT) method (Wu (2012)). We examine the effects of these adjustments on the estimation of intraday volatility using the Autoregressive Conditional Duration-Integrated Conditional Variance (ACD-ICV) method of Tse and Yang (2012). We find that daily volatility estimates are not sensitive to intraday periodicity adjustment. However, intraday volatility is found to have a weaker U-shaped volatility smile and a biased trough if intraday periodicity adjustment is not applied. In addition, adjustment taking account of trades with zero duration (multiple trades at the same time stamp) results in deeper intraday volatility smile.

1. Introduction

A well known problem in analyzing high-frequency financial data is the stylized fact of intraday periodicity: trading activities are usually higher at the beginning and close of the trading day than around lunch time. This trading pattern induces the average transaction duration to exhibit an inverted U-shape over the trading day. Andersen and Bollerslev (1997) point out that volatility over different intervals of the same calendar-time length at different times of the day may differ due to their differences in trading activities.

Recognizing the empirical fact of duration clustering, Engle and Russell (1998) introduce the Autoregressive Conditional Duration (ACD) model. They propose to correct for the intraday duration periodicity prior to fitting the ACD model to the data. Specifically, they apply the Duration Adjustment (DA) method to adjust for transaction duration as

$$\tilde{x}_{i+1} = \frac{x_{i+1}}{\phi(t_i)}, \tag{1}$$

where t_i is the calendar time of occurrence of the ith trade, $x_{i+1} = t_{i+1} - t_i$ is the duration of the $(i+1)$th trade in calendar time, $\tilde{x}_{i+1}$ is the diurnally adjusted duration of the $(i+1)$th trade and $\phi(\cdot)$ is the diurnal adjustment factor with its argument usually taken as the calendar time t_i.

Recently Wu (2012) proposes a new method to correct for intraday periodicity, which will be called the Time Transformation (TT) method in this paper. The theoretical underpinning of the TT method is that if there were no intraday differences in trading activities, we would expect the transactions to be evenly spread out throughout the trading day. Under the TT method a time-

* Corresponding author at: School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903, Singapore.
E-mail address: yktse@smu.edu.sg (Y.-K. Tse).

http://dx.doi.org/10.1016/j.jempfin.2014.04.004
0927-5398/

transformation function is determined using empirical data so that the transactions are evenly observed throughout the day under the transformed time.

To assess the empirical implications of these adjustment methods, we study their effects on daily and intraday volatility estimations. Following the method proposed by Tse and Yang (2012) for the estimation of high-frequency volatility, called the Autoregressive Conditional Duration-Integrated Conditional Variance (ACD-ICV) method, we examine the effects of the use of the DA and TT methods to adjust for intraday transaction durations on the estimation of intraday volatility. Our findings are as follows. First, for the estimation of daily volatility, whether or not the durations are diurnally adjusted makes little difference. Second, to estimate intraday volatility, correcting for intraday periodicity using either the DA or TT method produces more prominent U-shaped volatility smiles than not adjusting for intraday periodicity. Third, deeper intraday volatility smile is observed if the duration adjustment treats multiple trades with the same time stamp as separate trades (i.e., data with zero duration are retained). Fourth, intraday volatility smile has a biased trough if intraday periodicity adjustment is not applied.

The balance of this paper is as follows. Section 2 outlines the DA and TT methods. Section 3 summarizes the ACD-ICV method for estimating intra-day volatility. Section 4 reports the empirical results. Section 5 concludes.

2. Diurnal adjustment for intraday periodicity

The DA method involves adjusting the raw duration using a diurnal factor to obtain the diurnally adjusted duration, as given in Eq. (1). To specify the diurnal factor $\phi(\cdot)$ many studies use the regression method with linear spline or cubic spline method proposed by Engle and Russell (1998). While there are quite a few variations in the literature in the estimation of $\phi(\cdot)$, we follow the method proposed by Bauwens and Giot (2000), for which the focus is on estimating the expected duration conditional on the time of the day. The details of the procedure are summarized in Appendix A.1.

The DA method has some important drawbacks (see also Wu (2012)). First, the average duration in each interval is a local measure. There is a choice between taking longer intervals (so that there are more observations in each interval) versus shorter intervals (so that the knots are more precisely located). Second, given a raw duration, it is not clear which time point within the interval should be taken to evaluate the diurnal factor. As the diurnal-factor function takes an inverted U-shape, if the start time of the interval is taken for adjustment, which is the usual practice, the adjustment may be understated for trades before lunch time and overstated for trades after lunch time.[1] Hence, a systematic bias may be introduced in the diurnal adjustment.

We now briefly outline the TT method. Let t denote the calendar time and $\tilde{t}$ denote the corresponding diurnally transformed time (in seconds from the beginning of trade). In the context of the New York Stock Exchange (NYSE), as one trading day has 6.5 h (23,400 s), $0 \leq t \leq 23,400$. We denote $Q(t)$ as the empirical average proportion of trades in a day up to time t and define the diurnally transformed time by $\tilde{t} = 23,400Q(t)$. Conversely, given a diurnally transformed time $\tilde{t}$, the corresponding calendar time is $t = Q^{-1}\left(\tilde{t}/23,400\right)$, where $Q^{-1}(\cdot)$ is the inverse function of $Q(\cdot)$. Appendix A.2 provides more details for the computation of $Q(\cdot)$.

Given any two calendar-time points $t_i < t_j$, the diurnally adjusted duration between these two time points is $\tilde{t}_j - \tilde{t}_i = 23,400\left[Q(t_j) - Q(t_i)\right]$. Likewise, given any two diurnally transformed time points $\tilde{t}_i < \tilde{t}_j$, the corresponding duration in calendar time is

$$Q^{-1}\left(\frac{\tilde{t}_j}{23,400}\right) - Q^{-1}\left(\frac{\tilde{t}_i}{23,400}\right). \tag{2}$$

There are some advantages of the TT method over the DA method. First, the $Q(\cdot)$ function is easy to compute and it depends on all data in the sample. Second, the definition of diurnally adjusted duration is natural. This removes the ambiguity in the choice of the time point within the transaction interval for the application of the diurnal factor. Third, the switch between calendar time and diurnally adjusted time can be performed easily. This facilitates simulation using models in one measure of duration (e.g., ACD model for diurnally adjusted duration) to draw implications for the market in another measure of time (e.g., implications for the market in calendar time). Recently, Dionne et al. (2009) suggest using the simulation method to estimate the intraday value at risk (IVaR). While the time interval specified may be in calendar time, the duration model estimated may be for diurnally adjusted data. The TT method will be convenient to use as the calendar time and diurnally adjusted time can be easily converted from one another.

3. Diurnal adjustment and intraday volatility estimation

To examine the empirical implications of the methods of adjusting for intraday periodicity we consider the effects of these adjustments on the estimation of intraday volatility.[2] We use the ACD-ICV method for estimating high-frequency volatility

[1] The phenomenon of an inverted U-shape diurnal-factor function may be due to market participants' different trading purposes at different periods. Market participants tend to trade heavily when the market is open due to the opening auctions. They trade more heavily near the end of the day due to the closing effect, closing their positions before the market closes.

[2] Research on intraday movements of equity prices has recently attracted much interest due to its implications for market participants such as day traders and market makers, as illustrated by the works of Giot (2005), Dionne et al. (2009), and Liu and Tse (2013).