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Aggregational Gaussianity and barely infinite variance in financial returns

Antonios Antypas [a], Phoebe Koundouri [b], Nikolaos Kourogenis [c,*]

[a] *Department of Banking and Financial Management, University of Piraeus, Greece*
[b] *Department of International and European Economic Studies, Athens University of Economic and Business, Greece*
[c] *Department of Banking and Financial Management, University of Piraeus, Greece*

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ABSTRACT

This paper aims at reconciling two apparently contradictory empirical regularities of financial returns, namely, the fact that the empirical distribution of returns tends to normality as the frequency of observation decreases (aggregational Gaussianity) combined with the fact that the conditional variance of high frequency returns seems to have a (fractional) unit root, in which case the unconditional variance is infinite. We provide evidence that aggregational Gaussianity and infinite variance can coexist, provided that all the moments of the unconditional distribution whose order is less than two exist. The latter characterizes the case of Integrated and Fractionally Integrated GARCH processes. Finally, we discuss testing for aggregational Gaussianity under barely infinite variance. Our empirical motivation derives from commodity prices and stock indices, while our results are relevant for financial returns in general.

1. Introduction

One of the most important questions in the financial literature concerns the distribution of financial prices. Kendall (1953) was the first to notice that the empirical distributions of successive price changes deviate from normality mainly because they exhibit excess kurtosis. Mandelbrot (1963) suggested that the observed leptokurtosis reflects that the variance of commodity or stock price changes is infinite. Specifically, Mandelbrot observed that the logarithmic price changes (hereafter referred to as 'returns') within a specific period of time, is the sum of elementary returns, ξ_i, between transactions that occur during this period. He then assumed that the variance of returns is infinite, which in turn implies that the Central Limit Theorem is not applicable. As a result, the sum of ξ_i's converges not to the normal distribution, but instead, to a Stable Paretian distribution. The latter is leptokurtic and has infinite variance. An alternative explanation for the observed leptokurtosis in the empirical distributions of returns was offered by, among others, Clark (1973) and Blattberg and Gonedes (1974). These studies argue that transactions are not spread uniformly across time, which in turn implies that the underlying distribution of returns is a mixture of normals, therefore the finite-variance assumption is not sacrificed.

The two competing explanations for leptokurtosis mentioned above, bare different implications about the behavior of the distribution of returns as we move from higher (say daily) to lower (say monthly) frequencies of observations. In particular, as we move from higher to lower frequencies, the degree of leptokurtosis diminishes and the empirical distributions tend to approximate normality. This stylized fact, referred to as "Aggregational Gaussianity", can be accounted for only by the 'mixture of

* Corresponding author at: Department of Banking and Financial Management, University of Piraeus, 80 Karaoli and Dimitriou str., 18534 Piraeus, Greece. Tel.: +30 2104142142; fax: +30 2104142341.
E-mail addresses: nkourogenis@yahoo.com, nkourog@unipi.gr (N. Kourogenis).

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normals' explanation of leptokurtosis and not by the infinite-variance alternative. Indeed, if the daily returns follow a stable Paretian distribution with characteristic exponent equal to $a<2$, then, under general dependence assumptions, all the moments of order $s\geq a$ of the monthly (or annual) returns will be infinite.[1] Hence, the property of infinite variance, implied by the stable-Paretian hypothesis, cannot coincide with aggregational Gaussianity (see also Campbell et al., 1997, p. 19).

In late 1980s, when GARCH models emerged, the issue of the parallel existence of infinite variance and aggregational Gaussianity re-emerged. The estimation of GARCH models for commodity or stock returns seemed to suggest: (i) the presence of a unit root (or long memory) in the conditional variance, and (ii) the gradual declining of conditional heteroskedasticity and the associated leptokurtosis of the unconditional distribution as we move from higher to lower frequencies of observation (see Diebold, 1988; Drost and Nijman, 1993). The general class of ARCH (∞) models was introduced by Robinson (1991) and includes as special cases several well known conditionally heteroskedastic models, as the ARCH and GARCH. Baillie et al. (1996), introduced the Fractionally Integrated GARCH (FIGARCH) models which, including IGARCH, are also special cases of ARCH (∞) models (see Giraitis et al., 2009, for a discussion).[2]

Nelson (1990) proved the existence of a stationary solution of the IGARCH (1,1) model. However, the derivation of sufficient conditions for the strict stationarity of the general FIGARCH model was proved to be more difficult. For example, Kazakevičius and Leipus (2003) proved the existence of a strictly stationary solution for the Integrated ARCH (∞) model under a condition that excludes the case of FIGARCH, and Zaffaroni (2004) established necessary and sufficient conditions for the second order stationarity of ARCH (∞) models. Only recently, Douc et al. (2008) introduced a sufficient condition for the stationarity of several classes of ARCH (∞) models, and provided an example where this condition is satisfied by a FIGARCH process.

Overall, empirical studies seem to suggest the simultaneous presence of two seemingly contradictory facts: aggregational Gaussianity and infinite variance. This inconsistency between infinite variance and aggregational Gaussianity motivated some authors to argue that the evidence of long memory in the conditional variance was in fact spurious. It may arise either from structural breaks in the unconditional variance (see Diebold, 1986; Diebold and Lopez, 1995; Lamoureux and Lastrapes, 1990) or from regime switching of the parameters of the conditional variance (see Fong and See, 2001; Fornari and Mele, 1997, among others). Even in these cases, however, an infinite variance stationary model may arise. For example, Liu (2009) introduces the Integrated Markov Switching GARCH model, for which he proves stationarity and infinite variance. Liu's result implies that infinite variance can occur even if the conditional variance parameters in all-but-one regimes correspond to a finite variance model.

In this paper we aim at reconciling aggregational Gaussianity and infinite variance without bringing in question the FIGARCH specification. We show that coexistence of infinite variance and aggregational Gaussianity is possible, provided that all the moments of the unconditional distribution whose order is less than two, exist. This moment condition is satisfied in certain classes of ARCH (∞) processes, including FIGARCH, implying that a FIGARCH process is indeed a process with barely infinite variance (see Douc et al., 2008; Kourogenis and Pittis, 2008).

The paper is organized as follows: In Section 2 we present evidence indicating that the returns of three stock indices and nine commodities observed at high frequencies exhibit both leptokurtosis and long memory in the conditional variance. We also show that these effects tend to diminish as we move to lower frequencies. In Section 3 we explain why there is no paradox in admitting the simultaneous existence of aggregational Gaussianity and FIGARCH and discuss whether the mixing properties of a FIGARCH process conform to those assumed in the relevant limit theorems. In Section 4 we discuss testing for aggregational Gaussianity under infinite variance and present some additional empirical evidence supporting the coexistence of infinite variance and aggregational Gaussianity. The last section concludes the paper.

2. Empirical motivation: distributional characteristics of returns

The motivation for this paper derives from the changes observed in the distributional characteristics of financial returns, as we move from higher to lower frequencies. To empirically illustrate these changes, we use a dataset on spot prices of nine commodities obtained from S&P Goldman Sachs Commodity Indices (corn, soybeans, wheat, gold, silver, cattle, hogs, gasoline and copper) and three major stock indices (S&P 500, DAX 30 and NIKKEI 225). As in Baillie et al. (2007), we filter out the deterministic periodicities observed in the daily corn and soybean returns volatility by applying the Fourier Flexible Form (Gallant, 1981). As illustrated in the supplemental file of the paper (Fig. S1), all distributions are leptokurtic for the daily frequency, while the degree of leptokurtosis of the empirical distributions decreases as we move from daily to annual returns. Table 1 presents the median of the kurtosis coefficient, and the Jarque–Bera and Anderson–Darling test statistics for all returns under consideration and for all examined frequencies.[3] The evolution of the kurtosis coefficient suggests that when the frequency of the observations decreases distributions become less leptokurtic, while Jarque–Bera and Anderson–Darling statistics do not reject the null of normal distributions in low frequencies. The results are in agreement with other examples of similar behavior of financial returns, provided in the relevant literature (see, e.g., Campbell et al., 1997, ch. 1).

[1] Such dependence assumptions are not significantly restrictive. For example, they should exclude cases for which the returns are given by $R_{t+1}=q_{t+1}-R_t$, where q_{t+1} has finite moments of order $s>a$.

[2] Hereafter, the notation FIGARCH will correspond to the cases where the fractional order of integration is larger than zero. Specifically, we will consider IGARCH as a special case of FIGARCH.

[3] We present the median of the values for each statistic because we observe many similarities between the patterns observed in the asset returns under consideration. In particular, when we use annual returns, the values of the statistics approximate the values that correspond to the normal distribution.