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Application of cross-ratio in traffic accident reconstruction

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ABSTRACT

Nowadays, video recording devices such as CCTV, digital cameras, mobile phones and car video recorders are ubiquitous. It becomes more and more frequent that traffic accident scenarios are captured by such video recording devices in one form or another. The present study focuses on the direct extraction of vehicle speeds from video footages by cross-ratio, a well known invariant under specific conditions in projective geometry. The minimum requirements for an accurate and direct determination of vehicle speed by cross-ratio are 2 image frames of the video footage containing the subject vehicle which by and large moves along a straight path from one frame to the other and the time lapsed between the 2 image frames are known. With the aid of a calibrated Doppler radar, a control study has been carried out to validate the method and determine the baseline uncertainty. The validated method has been applied to a small number of real world cases and the results are promising. Experimental results indicate that it is possible to extend its application in speed determination by a car camera.

1. Introduction

Nowadays, it is very common to have CCTVs installed in banks, stores and public areas as a deterrent to crime. The use of video recording devices as means for crime fighting has a long history. Application has been found in many areas of forensic science. Wooller [1] wrote a guide to the effectiveness and evidential usage of Digital CCTV. Hugemann [2] gave a technical account in digital photo and video editing in accident reconstruction. Compton et al. [3] reported some pioneer work on the determination of vehicle speed from CCTV images. They located the concerned vehicle(s) in different frames of the CCTV and calculated the vehicle speed(s) by a simple distance–time calculation. The location of the concerned vehicle(s) was made either through observation by independent assessors or comparison with some reference points.

Use of car video recorders, commonly known as car cameras or dash cameras, among drivers also became popular to capture video for evidence in the event of a car accident. Footage from a car camera not only provides an unbiased account of events but also crucial speed information. Unlike CCTV cameras, which are usually stationary and capture moving objects, car cameras are mostly moving whilst recording. By making reference to stationary objects along a straight path of travel, which the car camera moved through, the speed of the moving car as well as other moving car could also be determined.

This paper proposes a method, by an application of cross-ratio, to determine the speed of a vehicle directly from the image sequences of a video footage from stationary camera without any reference to the physical position of the car at the scene. The possibility of extending its application to a moving car camera was also explored.

2. Method

2.1. Image frame

Each image frame captured by a recording device is a 2-dimensional representation of a 3-dimensional space. Each 3-dimensional position in the real world captured in a frame has mapped to a 2-dimensional point in the frame. The mapping, or transformation, from a 3-dimensional space to a 2-dimensional space, follows projective geometry principles. Fig. 1 depicts the mapping of a line ABCD in the 3-dimensional real world onto a 2-dimensional space of the image frame.

2.2. Collinearity

If four points *A, B, C* and *D* are collinear in the 3-dimensional real world and are transformed to a 2-dimensional space by projection, the respective points *A'*, *B'*, *C'* and *D'* in the 2-dimensional space are

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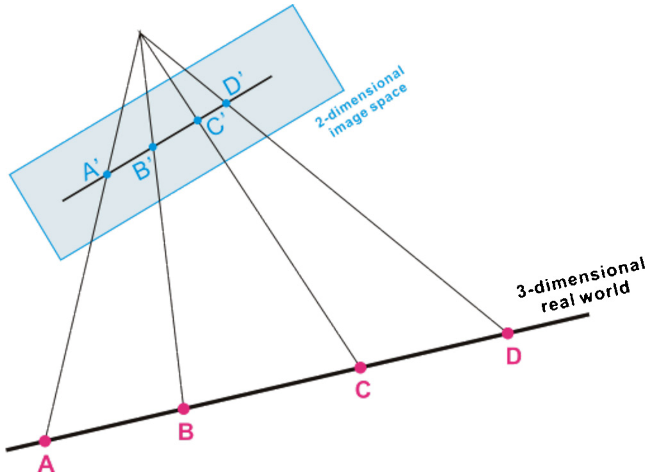

Fig. 1. Projective transformation from 3D space to 2D space.

also collinear. Collinearity of any points will not be changed upon projective transformation and is therefore a projective invariant [4]. In trivial terms, a straight line in the real world remains to be a straight line in an image.

2.3. Cross-ratio

Cross-ratio $(ABCD)$ of four collinear points A, B, C and D as depicted in Fig. 1, by definition is:

$$(ABCD) = \frac{|AC|/|BC|}{|AD|/|BD|} \tag{1}$$

The cross-ratio of any four points of a line is preserved by projective transformations [4] and is a projective invariant. In Fig. 1, the four collinear points A, B, C, and D in the real world are projected to A', B', C' and D', respectively in the image space. If $(A'B'C'D')$ denotes the cross-ratio of the four projected points A'–D', then $(ABCD)$ and $(A'B'C'D')$ are equal [4].

$$(ABCD) = (A'B'C'D') \tag{2}$$

Fig. 2 shows a car having travelled along a straight path at a speed v from time T_1 to time T_2.

A and B are the positions of the front and rear wheels on the path at time T_2. C and D are the positions of the front and wheel wheels on the path at time T_1. d is the distance travelled by the car and l is the wheelbase of the car. The four points A–D form a straight line and its cross-ratio $(ABCD)$ could be expressed in terms of d and l as below.

$$(ABCD) = \frac{|AC|/|BC|}{|AD|/|BD|} = \frac{d/(d-l)}{(d+l)/d}$$

$$(ABCD) = \frac{d^2}{d^2 - l^2}$$

Rearranging, we have

$$d = \pm\sqrt{\frac{(ABCD)}{[(ABCD)-1]}l^2} \tag{3}$$

The cross-ratio $(ABCD)$ is an imaginary parameter in the 3-dimensional real world because the two pairs of points A, B and C, D do not co-exist. If the movement of the car is captured in a video footage which is a time-ordered sequence of images, it is possible to extract the two image frames at T_1 and T_2 and construct a virtual 2-dimensional image space which contains all respective projected points of points A–D. From the virtual 2-dimensional image space, the cross-ratio $(ABCD)$ could be determined indirectly.

Similarly, a car, equipped with a car camera, is travelling along a straight path at a steady speed and the car camera captures two images at T_3 (Fig. 3) and T_4 (Fig. 4). Any road fixtures, such as the signposts on the left side of the carriageway in Figs. 3 and 4, would appear to the car camera (or a passenger inside the car) to be travelling at the same speed as the car but in opposite direction as demonstrated by their composite image in Fig. 5. Again, a constructed virtual 2-dimensional image space could be used for calculation of distance travelled by the signposts (distance travelled by the car) as the cross-ratio of any selected objects is also preserved.

2.4. Construction of virtual 2D image space

Figs. 6 and 7 capture the respective positions of a car which is travelling along a straight path at two different instances. The centres of the left front and rear wheels are labelled as C', D' in Fig. 6 and A', B' in Fig. 7, respectively.

Figs. 6 and 7 are imported into the photo editing software such as Photoshop® as overlapping layers to give a composite image. The properties of the layers of the composite image are manipulated such that the car images captured at the two separate instances are clearly visible in one view as shown in Fig. 8. It is of prime importance that the positions of the car with respect to the scene are not disturbed in the manipulation process.

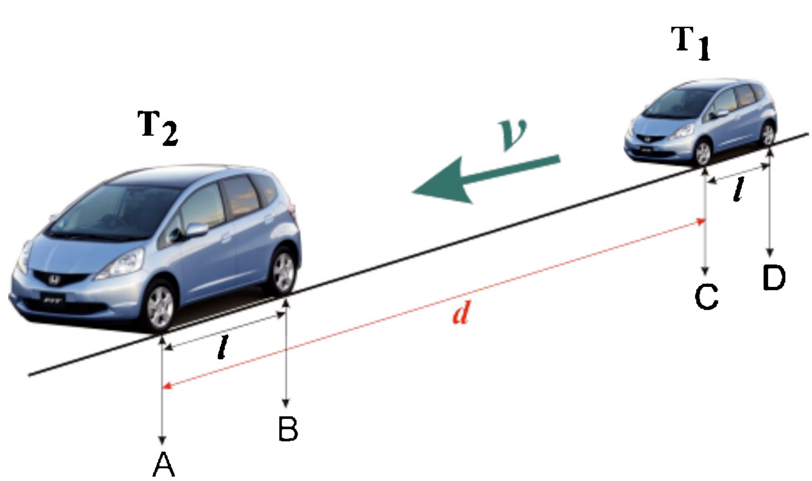

Fig. 2. A car travels along a straight path at a speed v.