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Regression-based estimation of dynamic asset pricing models ☆

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We propose regression-based estimators for beta representations of dynamic asset pricing models with an affine pricing kernel specification. We allow for state variables that are cross-sectional pricing factors, forecasting variables for the price of risk, and factors that are both. The estimators explicitly allow for time-varying prices of risk, time-varying betas, and serially dependent pricing factors. Our approach nests the Fama-MacBeth two-pass estimator as a special case. We provide asymptotic multistage standard errors necessary to conduct inference for asset pricing tests. We illustrate our new estimators in an application to the joint pricing of stocks and bonds. The application features strongly time-varying, highly significant prices of risk that are found to be quantitatively more important than time-varying betas in reducing pricing errors.

1. Introduction

Overwhelming evidence exists that risk premiums vary over time (Campbell and Shiller, 1988; Cochrane, 2011). Yet, widely used empirical asset pricing methods such as Fama and MacBeth (1973) two-pass regressions rely on the assumption that prices of risk are constant.

This paper proposes regression-based estimators for dynamic asset pricing models (DAPMs) with time-varying prices of risk. The estimators and associated standard errors are computationally as simple as Fama-MacBeth regressions, but they explicitly provide estimates of time-varying prices of risk, as well as estimates of the associated state variable dynamics. Our model combines key assumptions of the dynamic asset pricing models from fixed income applications with the computational ease of Fama-MacBeth regressions that are popular in empirical equity market research. The setup can also be viewed as a

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reduced form representation of dynamic macro-finance models with time-varying prices of risk.

We distinguish three different types of aggregate state variables: risk factors, price of risk factors, and factors that are both. By risk factors, we refer to variables that are significant factors for the cross section of asset returns, i.e. they have nonzero betas. By price of risk factors, we refer to variables that significantly forecast the time series variation of excess returns but do not necessarily have nonzero betas. Prices of risk are assumed to be affine functions of price of risk factors. We show that by introducing this risk price specification into generic asset pricing models, one can derive simple regression-based estimators for all model parameters that are consistent and asymptotically normal under mild conditions.

Our baseline estimator is a three-step regression that can be described as follows. In the first step, shocks to the state variables are obtained from a time series vector autoregression (VAR). In the second step, asset returns are regressed in the time series on lagged price of risk factors and the contemporaneous innovations to the cross sectional pricing factors, generating predictive slopes and risk betas for each test asset. In the third step, price of risk parameters are obtained by regressing the constant and the predictive slopes from the time series regression on the betas cross-sectionally. We give asymptotic variance formulas that allow for conditional heteroskedasticity and correct for the additional estimation uncertainty arising from using generated regressors.

We show that this three-step estimator coincides with the Fama-MacBeth estimator when two conditions are met. First, state variables have to be uncorrelated across time. Second, prices of risk have to be constant. Our approach can thus be viewed as a dynamic version of the Fama-MacBeth estimator, nesting the popular unconditional estimator as a special case.

We also introduce an additional (quasi-) maximum likelihood estimator (QMLE). This estimator is replacing the third regression step with a simple eigenvalue decomposition. The QMLE is asymptotically equivalent to the three step regression estimator even in the case of conditional heteroskedasticity in the return errors. We show that in our model generalized method of moments (GMM) and minimum distance (MD) estimation are exactly equivalent and that the QMLE is a special case of this more general class of estimation approaches for certain choices of weighting matrix.

While our main results are extensions of classic results in the cross-sectional pricing literature to a dynamic setting, we provide new interpretations of results in the model when prices of risk are constant. For example, the equivalence between GMM and MD estimation implies that the cross sectional T^2 statistic of Shanken (1985) could be directly interpreted as a J-test for the moment restrictions of the static model.

We also extend the three-step regression estimator to the case where betas and the parameters in the vector autoregression of the state variables are time-varying. We assume that these parameters evolve smoothly over time and estimate them using a kernel regression approach pioneered by Robinson (1989). Kernel regressions have the appealing feature of nesting least squares rolling window regressions which are often used in the empirical literature (see, for example, Fama and French, 1997; Lewellen and Nagel, 2006; among many others). In our implementation, however, we use a Gaussian kernel estimator with data-driven bandwidth choice following Ang and Kristensen (2012).

The affine price of risk specification we use closely resembles affine term structure models.[1] Our approach thus lends itself to asset pricing applications across different asset classes. We present an empirical application for the cross section of size-sorted equity portfolios and maturity-sorted Treasury portfolios. We show that a parsimonious model with two pricing factors, two price of risk factors, and one factor that serves both roles fits this cross section of test assets very well on average, while, at the same time, giving rise to strongly significant time variation in risk premiums. We further find that allowing for time variation in prices of risk is more important than modeling time variation in factor risk exposures in terms of minimizing squared pricing errors of the model. In our application, traditional estimation approaches such as the one by Fama and MacBeth (1973) and Ferson and Harvey (1991) imply substantially larger pricing errors than the estimators we propose.

The remainder of the paper is organized as follows. Section 2 provides a discussion of the contribution of this paper to the existing literature. We present the dynamic asset pricing model in Section 3. We discuss estimation and inference when betas are assumed to be constant in Section 4. In Section 4.1, we formally present the link of the dynamic asset pricing estimator to the static Fama-MacBeth estimator, and we explain the contributions of our results to the existing literature in detail. In Section 5, we derive the corresponding estimator under the assumption that betas vary over time. We illustrate our estimators in an empirical application in Section 6. Section 7 concludes.

2. Related literature

Our approach can be seen as a generalization of the static Fama and MacBeth (1973) cross sectional asset pricing approach to dynamic asset pricing models. The empirical applications of the static Fama-MacBeth approach are too numerous to list, but some of the seminal works are Chen, Roll, and Ross (1986) and Fama and French (1992).

Some previous authors have extended the Fama-MacBeth approach to conditional asset pricing models. Ferson and Harvey (1991) use period-by-period Fama-MacBeth regressions to obtain estimates of time-varying market prices of risk, which they then regress on lagged conditioning variables. They find evidence for predictable variation in prices of risk and associate most of the

[1] For regression-based approaches to term structure models featuring an exponentially affine pricing kernel, see Adrian, Crump, and Moench (2013) and Abrahams, Adrian, Crump, and Moench (2014).