Journal of Financial Economics 117 (2015) 470–488

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Incremental variables and the investment opportunity set ☆

Eugene F. Fama [a], Kenneth R. French [b,*]

[a] Booth School of Business, University of Chicago, Chicago, IL 60637, USA
[b] Amos Tuck School of Business, Dartmouth College, Hanover, NH 03750, USA

ARTICLE INFO

Article history:
Received 24 September 2013
Received in revised form
13 October 2014
Accepted 24 December 2014
Available online 13 May 2015

JEL classification:
G11
G12

Keywords:
Incremental variables
Investment opportunity set
Portfolio returns
Variable attenuation

ABSTRACT

Variables with strong marginal explanatory power in cross-section asset pricing regressions typically show less power to produce increments to average portfolio returns, for two reasons. (1) Adding an explanatory variable can attenuate the slopes in a regression. (2) Adding a variable with marginal explanatory power always attenuates the values of other explanatory variables in the extremes of a regression's fitted values. Without a restriction on portfolio weights, the maximum Sharpe ratios in the GRS statistic of Gibbons, Ross, and Shanken (1989) provide little information about an incremental variable's impact on the portfolio opportunity set.

1. Introduction

Many asset pricing papers identify individual variables (e.g., size, the book-to-market ratio, momentum, accruals, net share issues – the list is long) that are related to average returns. A variable's importance is often judged by the spread it produces in average returns. The emphasis is typically on the extreme quintiles or deciles from a sort on the variable, often with eye-popping results. Given the many patterns already identified in the cross section of average returns, however, a new variable's univariate average return spread almost certainly overstates its incremental contribution to the average return spread for portfolios formed using multiple forecasting variables.

Our goal is to explain a variable's incremental contribution to the average return spread from a multivariate forecast. We take the perspective of a researcher who uses estimates of expected returns from Fama-MacBeth (FM, 1973) cross-section regressions to sort stocks into portfolios. A variable's incremental impact on the spread in average returns from a multivariate regression is usually smaller than the spread from a univariate sort for two related reasons. First, a new explanatory variable often attenuates the slopes of variables already in the regression. Slope attenuation occurs in a bivariate regression, for example, when the explanatory variables are positively correlated and their slopes have the same sign or when they are negatively correlated and their slopes have opposite signs. The result is a reduction in the incremental variable's impact on the expected return spread.

The second driver is variable attenuation. Adding a variable with marginal explanatory power always shrinks the values of other explanatory variables in the extremes of a regression's fitted values. As a result, getting the expected return benefits of an additional variable almost always involves losing some of the gains from variables

☆ Eugene F. Fama and Kenneth R. French are stockholders in, board members of, and consultants to Dimensional Fund Advisors. We thank participants at the 2012 Finance Down Under Conference and the Norwegian School of Economics and Business Administration Finance Seminar, and an anonymous referee for helpful comments.

* Corresponding author.
E-mail address: Kenneth.R.French@tuck.dartmouth.edu (K.R. French).

http://dx.doi.org/10.1016/j.jfineco.2015.05.001
0304-405X/

already in the mix. The intuition is straightforward. Sorting on the fitted values from a one-variable regression is equivalent to sorting on the variable itself, so the high and low expected return portfolios from the one-variable regression maximize the spread in the explanatory variable. A new variable can increase the expected return spread only by replacing stocks in the extreme portfolios, and the turnover must reduce the spread in the original variable. This logic, which generalizes to multivariate regressions, says a larger increase in the expected return spread necessarily implies more turnover and more attenuation of the original variables.

Since Fama-MacBeth *t*-statistics measure whether specific variables are related to the cross section of expected returns, one might be tempted to argue that our focus on average return spreads is misplaced. However, a variable can have substantial power to describe variation in average returns when other variables are held constant (the thought experiment captured by the *t*-statistics for average multivariate FM regression slopes) but much less power to produce increments to average return spreads when faced with the joint variation of all variables in a model (the experiment implied by sorts on regression fitted values). To understand how variables combine to produce the expected returns available to investors, it is better to examine spreads in average returns on portfolios formed using regression forecasts with and without the additional explanatory variable.

Two earlier papers use regression fitted values to provide evidence on incremental returns. Fama and French (2006) examine average return spreads for portfolios that split stocks on the median fitted value from cross-section regressions. They comment that adding variables with strong marginal explanatory power in FM regressions (large *t*-statistics for average slopes) produces only modest improvements in average return spreads, but there is a suspicion that this is due to splitting stocks into just two portfolios. Lewellen (2011) finds, however, that adding explanatory variables to his cross-section regressions produces modest improvements in average return spreads for decile portfolios.

Lewellen (2011) tests models that use three (market capitalization, which we call *Size;* the book-to-market ratio *B/M;* and momentum), seven (first three, plus stock issues, accruals, profitability, and asset growth), and 15 (first seven plus eight more) forecasting variables. It is not surprising that the jump from seven to 15 variables produces little gain in average return spreads since many of the additional variables have weak explanatory power in the regressions. The small improvement in the jump from three to seven variables is more surprising because the additional variables show strong explanatory power. Similar comments apply to Fama and French (2006), whose regression models contain two, five, seven, and nine explanatory variables.

Fama and French (2006) and Lewellen (2011) do not explain why variables that have strong explanatory power when added to cross-section regressions produce only small increments to average return spreads in sorts on regression fitted values. That is the task addressed here.

We emphasize that we do not provide an exhaustive examination of how the panoply of average return variables identified in the literature combine to produce the cross section of expected returns. Our more limited goal is to illuminate the two general forces – slope and variable attenuation – that explain why variables that produce large average return spreads in univariate sorts and have strong marginal explanatory power in multivariate cross-section return regressions typically have less impact on the investment opportunity set when faced with competition from other variables. To keep the story simple, we focus on the three explanatory variables of Lewellen's first model (*Size*, *B/M*, and momentum), which are among the premier candidates from the literature.

For more perspective on whether an incremental variable improves an investor's opportunity set, we turn to the GRS statistic of Gibbons, Ross, and Shanken (1989). The GRS statistic is commonly used in time-series tests of asset pricing models. The GRS test asks whether the expected returns on a model's factor portfolios span the expected returns on a broader set of test portfolios.

We use the GRS statistic in an unusual way, to test whether quintile portfolios formed by sorting on the fitted values from two-variable regressions span the opportunity set obtained with quintile portfolios built from regressions that add a third variable. The arguments in GRS (1989) say that this is equivalent to testing whether the highest Sharpe ratio one can construct using the quintile portfolios from both the two- and three-variable regressions is reliably higher than the highest Sharpe ratio one can construct with just the quintiles from the two-variable regression.

We find that adding a third variable produces a large increase in the maximum Sharpe ratio and a strong GRS rejection of the hypothesis that the quintile portfolios formed by sorting on the fitted values from a two-variable regression span the opportunity set that can be obtained with three variables. This evidence is, however, of little use to investors. The portfolios that produce the maximum Sharpe ratios in the GRS statistic typically involve unrealistic leverage, with short positions that commonly exceed one hundred times the portfolio investment. When we examine the maximum Sharpe ratios that can be obtained in the absence of short selling (the opportunity set of a long-only investor), the increments to the ratio obtained by adding the third variable are typically small, often trivial.

We proceed as follows. Section 2 uses a simple model of the return generating process to explain how slope and variable attenuation combine to determine a variable's incremental contribution to a multivariate forecast of returns. Sections 3 and 4 illustrate our conclusions with regressions that use *Size*, *B/M*, and momentum to forecast returns. Section 5 presents evidence on the importance of short sales in the tangency portfolios at the heart of the GRS test and explores the impact of short sale constraints on maximum Sharpe ratios. A summary and conclusions are in Section 6. Appendix A provides additional empirical results and additional formal analysis of the model of Section 2.

2. Incremental return spreads: a simple model

We use fitted values from FM regressions of individual stock returns on characteristics to sort stocks into quintile