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On variance bounds for asset price changes

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ABSTRACT

In this paper, I consider variance bounds for stock price changes in a general setting that allows for ex-dividend stock prices, risk-averse investors, and exponentially growing dividends. I show that providing investors with more information about future dividends can either increase or decrease the variance of stock price changes, depending on key parameters, namely, those governing the properties of dividends and the stochastic discount factor. This finding contrasts with the results of Engel (2005), who shows that news about future dividends will always decrease the variance of stock price changes in a specialized setting with cum-dividend stock prices and risk-neutral investors.

1. Introduction

In theory, the price of a stock represents the market's consensus forecast of the discounted sum of future dividends that will accrue to the owner. If dividends are stationary, the variance of observed stock prices (the forecast) should be lower than the variance of discounted realized dividends (the object being forecasted). LeRoy and Porter (1981) and Shiller (1981) argue that this rationality principle appears to be violated in the case of U.S. stock prices.

West (1988) extends the analysis to allow for nonstationary dividends. He shows that the variance of *unexpected* changes in the stock price will decline if risk-neutral investors are given more information about future dividends. This is because rational investors will use any new information to improve the precision of their dividend forecasts, thereby reducing the variance of the forecast errors.[1]

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[1] However, West (1988, p. 41) includes the caveat that his result "may not extend immediately if logarithms or logarithmic differences are required to induce stationarity [of the dividend process]".

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Engel (2005) extends the analysis of West (1988) to consider the variance of *actual* changes in the stock price, i.e., $\Delta p_t \equiv p_t - p_{t-1}$, as opposed to unexpected changes. Engel's analysis allows the level of real dividends to evolve as an arithmetic random walk or a stationary stochastic process. Assuming that stock prices are "cum-dividend," he shows that the variance of Δp_t will decline if risk-neutral investors are given more information about future dividends. In particular, he proves analytically that $Var(\Delta p_t) \geq Var(\Delta p_t^*)$, where p_t^* is the stock price computed using perfect foresight about future dividends. Also assuming risk-neutral investors, LeRoy (1984) had previously demonstrated the result $Var(\Delta p_t) > Var(\Delta p_t^*)$ in a calibrated model where stock prices are "ex-dividend." LeRoy assumes that dividends are stationary but highly persistent.[2] The perfect foresight case is the same benchmark used by Shiller (1981) to argue that the opposite variance ordering prevails when it comes to the price level, i.e., $Var(p_t) \leq Var(p_t^*)$.

The foregoing results have been interpreted in the literature to imply that news about future cash flows must decrease the volatility of asset price changes. For example, Engel (2014, p. 464) states "… the variance of changes in the asset price falls with more information…[N]ews can account for a high variance in the real exchange rate, but not for a high variance in the change in the real exchange rate".

In this paper, I expand the modeling framework of Engel (2005) to consider the more standard setup of "ex-dividend" stock prices, risk-averse investors, and exponentially growing dividends. I consider three different information sets, labeled I_t, I_t^o, and I_t^*, which contain progressively increasing amounts of information, i.e., $I_t \subseteq I_t^o \subseteq I_t^*$. Under set I_t, the investor can observe current and past dividend realizations and thereby identify the law of motion for dividends. Set I_t^o, denoted by the superscript "o," allows investors to have one-period foresight about dividends. This setup captures the possibility that investors may have some news that allows them to accurately forecast dividends over the near-term.[3] Finally, set I_t^*, denoted by the superscript "$*$," provides the maximum amount of investor information, corresponding to perfect knowledge about the entire stream of past and future dividends. The symbols p_t, p_t^o, and p_t^* represent the equilibrium prices under the three information sets.

I show that providing investors with more information about future dividends can either increase or decrease the variance of stock price changes. In particular, I show that the direction of the price change variance inequality can be reversed, depending on the values assigned to some key parameters of the model. These include a dividend persistence parameter ρ, the investor's subjective time discount factor β, and the coefficient of relative risk aversion α.

Following Engel (2005), I initially consider an economy where the representative investor is risk-neutral ($\alpha = 0$) and dividends follow an arithmetic AR(1) process that allows for a unit root as a special case. When observed stock prices are cum-dividend, I recover a variance ordering consistent with Engel's theoretical propositions, namely, $Var(\Delta p_t) \geq Var(\Delta p_t^o) \geq Var(\Delta p_t^*)$. However when observed stock prices are ex-dividend, I show that $Var(\Delta p_t)$ can be greater or less than $Var(\Delta p_t^*)$, depending on the values of ρ and β. The two variance statistics are exactly equal when the parameters satisfy the condition $\rho(1+\beta) = 1$. For a typical model calibration where dividends are a close to a random walk and the discount factor is close to unity, we have $\rho(1+\beta) > 1$, which in turn yields $Var(\Delta p_t) > Var(\Delta p_t^*)$, thus confirming the numerical results obtained by LeRoy (1984). LeRoy's model calibration satisfies the condition $\rho(1+\beta) > 1$. Engel's cum-dividend model can be interpreted as imposing the parameter restriction $\rho\beta \simeq 1$ such that the condition $\rho(1+\beta) > 1$ is once again satisfied. However, if dividends are less persistent or the future is more heavily discounted such that $\rho(1+\beta) < 1$, then the variance inequality will be reversed, yielding $Var(\Delta p_t) < Var(\Delta p_t^*)$. Similarly, I show that variance ordering for Δp_t and Δp_t^o can be reversed if stock prices are ex-dividend and $\rho < 1$.

The explanation for the variance ordering reversals is linked to the discounting mechanism. The parameters ρ and β both affect the degree to which future dividend innovations influence the perfect foresight price p_t^* via discounting from the future to the present. The future dividend innovations have no effect on p_t because the expected value of future innovations is zero. When dividends are highly

[2] The result in LeRoy (1984) is further discussed by Gilles and LeRoy (1991, p. 771).

[3] Information set I_t^o connects to recent research on business cycles that focuses on "news shocks" as an important quantitative source of economic fluctuations (e.g., Barsky and Sims, 2011).