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Physico-mathematical modeling of crossflow filtration

A.S. Ferreira [a], G. Massarani [b,*]

[a] Centro de Pesquisas e Desenvolvimento/Petrobras, Cidade Universitária Q. 7, 21949-900 Rio de Janeiro, RJ, Brazil
[b] PEQ/COPPE, Universidade Federal do Rio de Janeiro, Caixa Postal 689502, 21945-970 Rio de Janeiro, RJ, Brazil

Abstract

A phenomenological model that allows the correlation of pressure fields, filtration rate and cake thickness in the process that results from the axial flow of a suspension in a duct that is provided with a porous wall is presented. The model is based in the motion and the continuity equations for the phases and incorporates the following constitutive information:

- Rheological properties of the suspension and of the fluid that makes up this suspension.
- Rheological properties of the cake including the effects of the compressibility and the relation between the shear stress of mobilization and the normal stress associated to the Coulomb criterion in the stabilization of the thickness of the cake.
- Properties of the filter medium.

The experiments performed with an aqueous suspension of calcium carbonate (average particle diameter in the order of 0.5 μm) confirms the common knowledge that the properties of the cake depend on the mode of filtration, making it of capital importance to perform specific assays for each case that is studied.

Keywords: Physico-mathematical modeling; Crossflow filtration; Non-Newtonian

1. Introduction

Crossflow filtration results from the axial flow of a suspension through a duct which walls are made of filter material, leading to the formation of a deposit on the filter surface and the production of a filtrate that percolates through it. Characteristically, the flow of filtrates drops with time, and may stabilize, in a longer operation, as a consequence of the action of the mechanisms that limit the growth of the cake.

Crossflow filtration is used in the industry for the clarification of effluents and in the concentration of suspensions in a wide range of applications that employ the technology of membranes [1].

The operation of drilling and preparation of inclined oil wells involves essentially the same phenomena that prevail in the crossflow filtration with micro-membranes. It is the axial flow of a non-Newtonian suspension in the annular space between a cylinder in rotation and the formation of petroleum. In this situation, the properties and the thickness of the deposit that is formed allow for the control of the damaging invasion of the drilling fluid in the petroleum formation [2].

This work is restricted to the study of the crossflow filtration that results from the axial flow of a suspension in a hose that was built with the filter medium used in industrial filtration. The modeling is made based on the continuity and motion equations for the phases, and has as its objective to establish the relation between the pressure fields, the flow of filtrate, and the thickness of the cake along the process [3]. The model considers the period of cake growing, with the attendant reduction in the rate of filtration, and the stage in which the thickness of the cake and the flow of filtrate may stabilize in a longer operation, in which the filter works as a thickener.

The constitutive information aggregated to the proposed model are to be established, whenever it is possible, in assays that do not use the results from crossflow filtration itself: rheometry for the survey of the properties of the suspension and of the fluid that makes up this suspension, dead-end filtration for the characterization of the cake and of the filter

* Corresponding author. Tel.: +55 21 2562 8345; fax: +55 21 2562 8300.
E-mail address: gmassa@peq.coppe.ufrj.br (G. Massarani).

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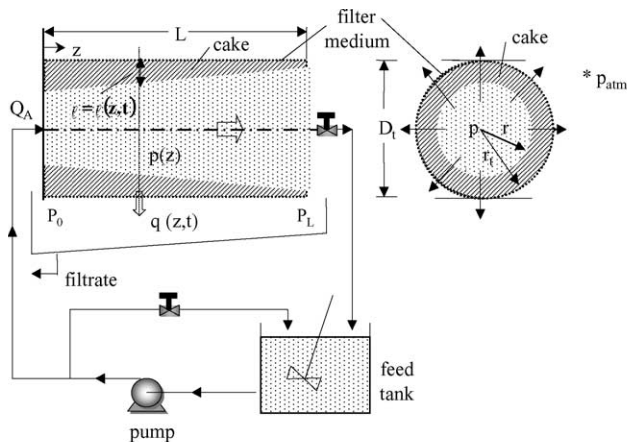

Fig. 1. Rig flow diagram.

medium, assay for establishing the relation between the normal and shear stresses of mobilization for the cake (direct shear test). This last assay is associated to the criterion of Coulomb for the stabilization of the cake.

Taking into consideration the probable differences between the mechanisms of formation of the cake in the modes of filtration, it is recommended that the cake be also characterized by means of assays in the crossflow filtration itself.

2. Physico-mathematical modeling

The modeling of the crossflow filtration that is proposed in this paper corresponds to the situation which scheme is showed in Fig. 1, in which the filtrate is weighed and discarded, and the suspension is returned to the feeding tank. During the stage of cake formation, the concentration of solids is kept constant; the beginning of the thickening of the suspension signals the start of the process of stabilization of the cake.

2.1. Flow of the suspension in the duct with porous wall

The pressure field that is established by the flow of the suspension through the tube may be uncoupled from the process of filtration when the permeability of the porous wall (filter medium + cake) is reduced. This situation widely prevails in the crossflow filtration.

The relation between the pressure drop and the flow rate in the axial flow is given by the following formula [4]:

$$-\frac{\Delta p}{L} = \frac{f V_M^2 \rho_M}{2 D_t} \quad (1)$$

$$\frac{p_0 - p(z)}{p_0 - p_L} = \frac{z}{L} \quad (2)$$

$$f = f\left(\frac{e}{D_t}, Re_M\right), \quad Re_M = \frac{D_t V_M \rho_M}{\mu_{ef}} \quad (3)$$

$$V_M = \frac{Q_A}{A_t}, \quad \rho_M = \varepsilon\rho_F + (1-\varepsilon)\rho_s \quad (4)$$

$$\mu_{ef} = \frac{S_M(\lambda^*)}{\lambda^*}, \quad \lambda^* = \frac{6.4 V_M}{D_t} \quad (5)$$

It is important to highlight that these results do not depend on the flow regime and that the effective viscosity μ_{ef} is calculated from the rheology of the suspension, with the knowledge of the characteristic rate of distension λ^*.

2.2. Filtration with cake formation

The initial step of crossflow filtration is characterized by the growth of the cake that is formed next to the filter medium, with the attendant reduction in the rate of filtration. These parameters vary with the position, as the pressure drop in the filtration

$$(\Delta p)_f = p(z) - p_a$$

reduces along the filter tube, as it is indicated by Eq. (2). Being the filtration performed in the circuit that is sketched in Fig. 1, the concentration of the suspension, c, is kept constant during the stage of cake growth.

The equations of conservation of mass for the phases and of the motion of the fluid, in the form of the Darcy equation, lead to the equation for crossflow filtration over a cylindrical surface [5,6]:

$$\frac{dt}{dv} = \frac{\mu_F}{(\Delta p)_f}\left\{\alpha\varepsilon_s\rho_s r_t \ln\frac{r_t}{\left[r_t^2 - \frac{2c\rho_F r_t v}{\varepsilon_s\rho_s}\right]^{1/2}} + R_m\right\} \quad (6)$$

where t is the time of filtration and v is the volume of filtrate per unit area of filtration,

$$v = \frac{1}{2\pi r_t}\frac{dV}{dz} \quad (7)$$

Being the crossflow filtration performed with a suspension that is comprised by particles within a distribution of size, the deposit of these particles is selective along the process that combines axial flow and percolation through the filter medium [3,7]. In other words, the resistivity α and the volumetric fraction of the solids ε_s, averages in each cross-section of the filter, vary not only with the filtration pressure at the location, but also with the structure of the cake that is deposited.

Being the cake compressible, the resistivity α and the volumetric fraction of the solids ε_s depend on the filtration pressure. R_m is the resistance of the filter medium.

The result given by Eq. (5) may be extended for the case in which the liquid that percolates through the cake and the filter medium is non-Newtonian. The effective viscosity may be calculated from the rheology of the liquid with the knowledge