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The role of wave propagation in hydrocyclone operations II: Wave propagation in the air–water interface of a conical hydrocyclone

E. Ovalle, F. Concha*

Department of Metallurgical Engineering, University of Concepción, Concepción, Chile

Abstract

The air-core plays an important role in the operation of a conical hydrocyclone. In the apex, the air-core increases in diameter with the possibility of reaching a size close to the apex diameter. In this case, roping may be induced. There is a range of values for the apex to vortex ratio where roping is possible. Using some simple physical models, this work shows that perturbations propagating through the air-core interface are amplified in the direction of the apex and may be responsible for the fluctuations of the underflow that are characteristic of roping.

Keywords: Hydrocyclone; Wave motion

1. Introduction

In previous work [10], we discussed the flow pattern in a conical hydrocyclone and obtained a numerical solution of the Navier–Stokes equation by the finite element method (FEM). In the present work, we focus on the existence and propagation of wave motion inside a conical hydrocyclone.

From a practical point of view, the principal interests are the efficiency of the classification process and in the operational stability of the hydrocyclone performance. The efficiency of classification is characterized by the sharpness of the separation, which can be visualized by the shape of the classification curve. In a numerical hydrocyclone model, the classification curve may be obtained by introducing particles in the flow field and following the trajectories. These trajectories are sensitive to the perturbations introduced by the instabilities of flow, and the presence of turbulence. In the presence of these phenomena, the classification curve is less sharp.

One feature that complicates the study of the flow in a conical hydrocyclone, is the presence of an air-core [1,4,6]. Electrical resistance tomography studies have shown that the air-core is not static [12], and some experimental observations show that the instabilities of the air-core could produce a degradation in the classification quality [9]. We think that perturbations induced by wave propagation in the flow may also influence the efficiency.

In this paper, the problem of generation and propagation of waves in a hydrocyclone is addressed. Through a hydraulic analogy, the study starts modelling the flow inside a conical hydrocyclone, where the centrifugal force is replaced by a gravitational force. In this context, it is possible to study some particular features of the flow such as the form adopted by the free surface, the propagation of superficial waves through the air–water interface, and the effect of the waves on the operation of a conical hydrocyclone. Next, we propose a possible mechanism for the generation of wave motions in a region adjacent to the air-core of a hydrocyclone.

2. Form of the free surface

The form of the free surface observed in a laboratory hydrocyclone is similar to that shown in Fig. 7. The surface is horizontal and flat (cylindrical in the actual case) and deformed in the outlet regions. As we will show, these deformations are evidence for the existence of wave-like motions in the region near to the air-core.

Consider, for example, the flow in a channel with a constant depth H [11] (see Fig. 1), where the flow has an upstream constant velocity U. If, in a certain region, a ramp is present

* Corresponding author.

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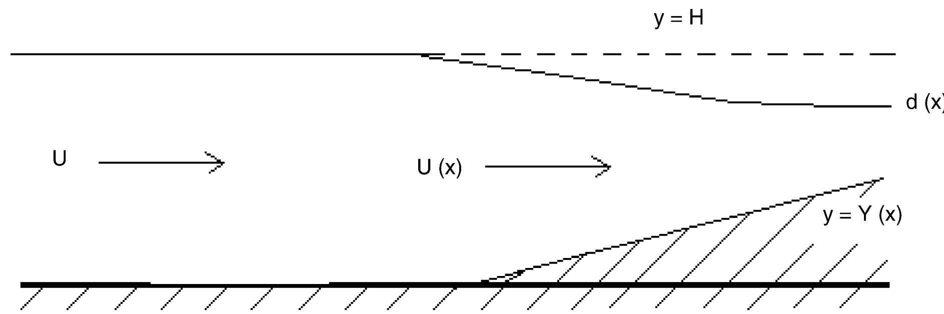

Fig. 1. Free-surface deformation in a channel.

in the form $y = Y(x)$, the height of the free surface will change from $y = H$ to $y(x) = H - d(x)$. The conservation of mass demands that:

$$UH = u(x)[H - d(x) - Y(x)] \quad (1)$$

On the other hand, momentum is conserved in the flow and may be expressed by the Bernoulli equation over a free surface streamline:[1]

$$\frac{1}{2}U^2 + gH + \frac{p}{\rho} = \frac{1}{2}u^2 + g(H - d) + \frac{p}{\rho} \quad (2)$$

Eliminating $u(x)$ from the above equations results in:

$$\frac{U^2H^2}{(H - d - Y)^2} = U^2 + 2gd \quad (3)$$

Since $d \ll H$, the LHS of Eq. (3) can be expanded in a rapidly convergent series. Truncating this series yields:

$$d(x) = \frac{Y(x)}{F^{-2} - 1} \quad (4)$$

where F is the Froude number defined by:

$$F = \frac{U}{\sqrt{gH}} \quad (5)$$

The flow is *subcritical* when $F < 1$ and *supercritical* when $F > 1$. In the first case, $d(x)$ has the same sign as $Y(x)$ and the free surface decreases its level downstream. The inverse occurs when $F > 1$. This change of regime, associated with the variation of the Froude number, determines how a perturbation is propagated through the flow. As is known in the shallow-water theory, the wave velocity of propagation c is proportional to $\sqrt{gH}$. Therefore, the Froude number, written as $F = \frac{U}{c}$, has a physical meaning similar to the Mach number in compressible flows. If the flow is supercritical, the velocity of the fluid is greater than the transmission of information about a change in the floor level; thus, the perturbation upstream has no effect on the flow downstream. In the opposite case, the information may advance faster than the flow, and the perturbations originated at the floor are reflected upstream, which are favorable conditions for the formation of waves in the flow.

In a conical hydrocyclone, the flow has a tangential component that is the same order of magnitude as the axial component except at the outlet region where the axial component is increased at the expense of the tangential component. Therefore, based on the observations of the free surface in the adjacent outlet regions, we assume that, *inside the hydrocyclone, the flow is in a subcritical state and, as a consequence, this condition is favorable for the existence of undulatory motions in the fluid*.

3. The hydraulic model

The work of Escudier et al. [8], who used a hydraulic analogy to study the change between subcritical and supercritical regimes of a swirly flow, inspired us to study the propagation of waves in the air-core of a hydrocyclone in a similar manner. The hydraulic model of the hydrocyclone consists of a launder with a ramp as shown in Fig. 2. The water enters through the bottom at the left side and overflows at the left and right of the upper part of the model.

The model is a bi-dimensional, Cartesian representation of a conical hydrocyclone geometry, as is shown in Fig. 2. In this representation, the centrifugal force is replaced by gravitational force, $-\rho\mathbf{g}$, in the vertical direction. The flow is modelled as inviscid and irrotational and the velocity is expressed in terms of a potential velocity $\mathbf{v} = \nabla\phi$. This approach has the following advantages: the boundary conditions can be chosen with a clear physical sense, in contrast with other formulations (for example, the stream-vorticity scheme). The second reason for this formulation is because the treatment of the undulatory phenomena is more direct.

In the model, the flow obeys: the continuity equation $\nabla^2\phi = 0$ and Bernoulli's equation of motion:

$$B(t) = \frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + gy + \frac{p}{\rho} \quad (6)$$

where p is the pressure field, $B(t)$ an arbitrary function of time, ρ the constant liquid density and g is the gravitational acceleration. Since the potential ϕ, is only unique up to a constant,

[1] In the hydrocyclone case, a centrifugal force $m\frac{v_r^2}{r}$ must be considered in Eq. (2).