Three-dimensional discrete element simulations in hoppers and silos

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Abstract

The initial fill and the subsequent discharge of containers containing granular material have been modeled using the discrete element method (DEM). In particular, the behavior of granular material in three dimensions is accounted for. The aim is to study the distributions of normal wall forces and pressures developed at the end of filling process (static state) and those developed during outflow from the containers (dynamic state). A hopper, a silo with a hopper bottom, and a flat-bottomed silo have been chosen for DE analysis. The granular material is modeled as a particle assembly consisting of mono-sized, spherical, cohesionless particles. Movement of individual particles during the outflow caused by gravity was also studied. The simulation results obtained in the present work are realistic and show clearly the advantages of DEM to understand the complex flow behavior of granular materials.
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1. Introduction

Containers with a variety of shapes are widely used in industry for storage of granular materials. At the end of the initial fill and during the subsequent discharge of a container, pressures acting on the walls and wall stresses can be predicted on the basis of both analytical and numerical calculations. Recently, two numerical methods have been widely used to study the mechanical behavior of granular materials in containers: the finite element method (FEM) and the discrete element method (DEM). The latter is a powerful numerical method, in which the motion of each individual particle is determined on the basis of all the forces acting upon it [1]. Contrary to continuum techniques, it simulates effects at particle level.

There are many reports about the investigations of container flows, but most of them deal with two-dimensional simulation and model particles as circular rods [2–4] for the purpose of simplifying mathematical treatment and reducing computer memory requirements. In most cases, analyses are performed by commercial simulation programs.

Two-dimensional DE analyses provide satisfactory results for several problems, but there are many phenomena, which require three-dimensional simulation. This has been emphasized, for example, in Dürr et al. [5]. Two- and three-dimensional discrete element simulations have been carried out for filling and discharge under gravity from model hoppers in Langston et al. [6]. The granular material is modeled as an assembly of non-cohesive discs or spheres. The results obtained are, in general, in very good agreement with established literature empirical predictions.

The main aim of the present study is to investigate the behavior of granular material after the initial fill (static state) and during the discharge (dynamic state) of different containers. For this purpose, a generally applicable three-dimensional DE computer program, developed by the authors, was used. The source code was written in C++ and compiled using the Visual C++ Version 6.0 compiler. The main advantages of the program are as follows: both the translational and the rotational motion of the particles are considered, the numerical integration of the Newton equations is performed by means of the efficient Feynman–Newton method, and, finally, it allows users to construct arbitrary geometry from planar walls with three or four vertexes.

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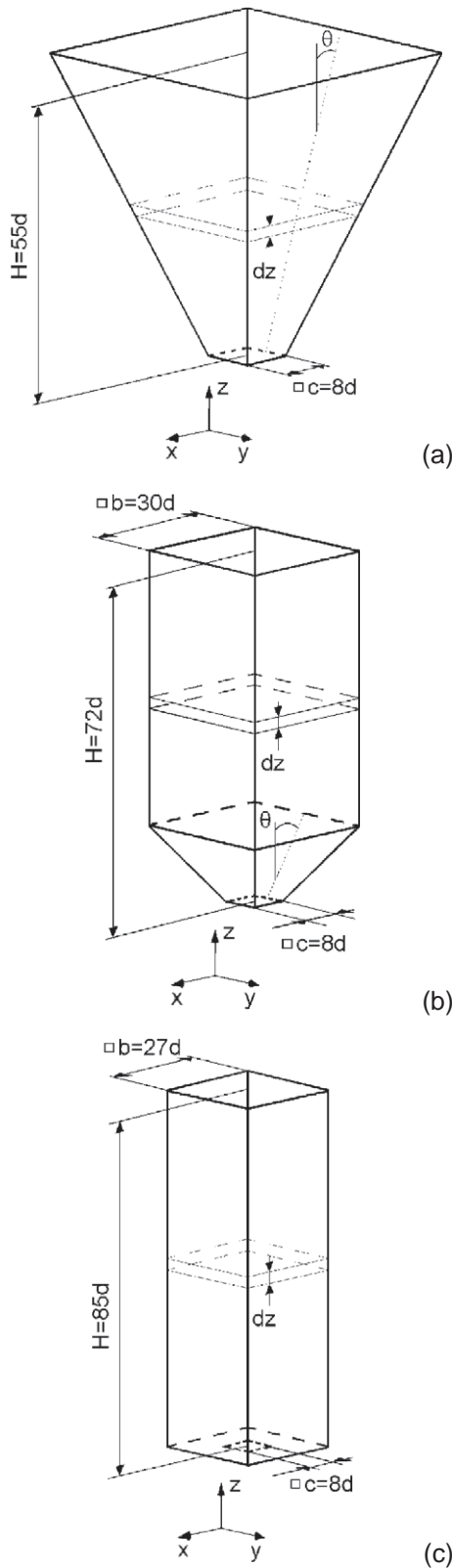 *E-mail address:* ebert@mv.uni-kl.de (F. Ebert).

Fig. 1. Geometry details of the containers studied: (a) pyramidal hopper, (b) silo with a hopper bottom, and (c) silo with a flat bottom (*d* is the particle diameter).

In a recent study, a system of 40,000 mono-sized cellulose acetate spheres of diameter $d = 6$ mm was considered, although the program developed allows us to investigate also the effects of polydisperse particles on the flow behavior. For numerical investigations, the following container types have been chosen: a pyramidal hopper (Fig. 1a), a silo with a hopper bottom (Fig. 1b), and a vertical-sided silo with a flat bottom (Fig. 1c). The hopper has an opening angle $\theta = 20°$ and a square cross section. The silo with a hopper bottom is $72d$ in height, has a square cross section with a size of $30d$, and a converging section with a hopper half angle of $\theta = 35°$. The flat-bottomed silo is $85d$ in height and has a square cross section with a size of $27d$. In all the cases studied, the size of the outlet is $8d$ and it has a square cross section. With these sizes, the height of the densely packed configuration of particles at rest is approximately the same in each case. The only external force acting on the system is the gravity ($g = 9.81$ m/s²). At the beginning of the simulations, the containers were filled by regularly located, non-overlapping particles. While particles had no initial velocity in the case of the hopper, a small random initial velocity was assigned to each particle in the other two cases. Subsequently, the systems were allowed to settle in order to obtain densely packed configurations before the container opening. At the end of settling, the static packing height of the particles is about 0.3 m in all the cases studied. The outlets were kept closed during the settling process. Based on preliminary calculations, a settling time of 1.5 s was specified for each simulation. At $t = 1.5$ s, the outlets were opened and, at the same time, the discharge of containers was started.

2. Simulation technique

Discrete element modeling of granular materials involves following the movement of all particles and the detection their collisions with other particles and with their environment. In order to calculate the motion of each particle, all the forces acting on a particle are summed and the resulting equations of motion are numerically integrated with a fixed time step. If the time step chosen is sufficiently small, it can be assumed that, during a single time step, the instantaneous motion of each particle is determined only by its immediate neighbors with which it is in contact [1]. This assumption greatly reduces computer memory requirement.

2.1. Equations of motion

Each particle has two types of motion: translational and rotational. These motions can be described by Newton's second law given by:

$$m\dot{v} = \sum_i (F_n + F_t) + mg \qquad (1)$$

$$\theta\dot{\omega} = \sum_i M \qquad (2)$$