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Modeling exchange rate dependence dynamics at different time horizons

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A B S T R A C T

Despite an extensive body of research, the best way to model the dependence of exchange rates remains an open question. In this paper we present a new approach which employs a flexible time-varying copula model. It allows the conditional correlation between exchange rates to be both time-varying and modeled independently from the marginal distributions. We introduce a dynamic specification for the correlation using the Fisher transformation. Applied to Euro/US dollar and Japanese Yen/US dollar, our results reveal a significantly time-varying correlation, dependent on the past return realizations. We find that a time-varying copula with the proposed correlation specification gives better results than alternative dynamic benchmark models. The dynamic copula model outperforms at six different time horizons, ranging from hourly to daily, confirming the model specification.

1. Introduction

In economics and finance multivariate problems are often of interest in areas like risk management, asset pricing, portfolio allocation and forecasting. There, the specification of financial variables' multivariate distributions is crucial for the computation of quantities related to their inter-dependence. The prime example is the linear correlation, the perfect measure of dependence for multivariate normal variables and to some extent for elliptical variables. However there is strong evidence that the univariate distributions of many financial variables are non-normal and significantly fat-tailed. This

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empirical fact often rules out the use of the multivariate normal distribution. In principle, there is no reason for different marginal variables to have the same degree of fat-tailedness or even to have univariate distributions of the same type. Moreover, most financial data exhibit skewness. This also questions the use of elliptical distributions in other contexts.

Financial time series are often modeled with GARCH type models. In the multivariate GARCH literature there exist several models, like CCC–GARCH, DVEC, matrix-diagonal GARCH, BEKK and principal components GARCH.[2]

In all these models the conditional multivariate distribution is Gaussian or Student-*t*. An alternative approach to overcome that distributional constraint is to use copula–GARCH models; see Jondeau and Rockinger (2006), Fortin and Kuzmics (2002), Patton (2006a,b).[3] Copula-based models allow complete freedom to combine different conditional marginal distributions in a dependence structure.

By construction, the conditional variance of the univariate distributions in a GARCH model is time-varying. Additionally, in the copula–GARCH model the parameters of the dependence structure can also be time-varying. This is potentially useful as there is evidence that asset's dependence is time-varying, a fact that has raised considerable interest in the dynamic behavior of correlation between different risks as a function of time; see for instance Boyer et al. (1999), Engle (2009), Longin and Solnik (2001), Loretan and English (2000) and Loretan and Phillips (1994). Because of the fundamental importance of the notion of linear correlation in finance and insurance, such dynamics may have a non-trivial impact on the pricing and hedging of underlying instruments, or on the risk measurement of such positions. As a consequence, a systematic modeling of the dynamic behavior of the dependence structure underlying multivariate variables is of considerable importance. This can be achieved with time-varying copula–GARCH models.

In the present article we present a time-varying copula–GARCH model. We propose a specification for the dynamics of the dependence parameter using the Fisher transformation in order to model the dynamic dependence between Euro and Japanese Yen versus U.S. Dollar exchange rates. With copula–GARCH models we allow for possibly different fat-tailed univariate distributions. We look for the most appropriate copula family for both time-invariant and time-varying copula models.

We compare the proposed model with two competing models: the dynamics proposed by Tse and Tsui (2002) and the BEKK model introduced by Engle and Kroner (1995). We evaluate the estimated dependence paths by comparing them with two benchmark non-parametric measures of dependence: the realized correlation and Kendall's tau. Finally we investigate six time horizons ranging from hourly to daily. Our goal is to examine the consistency of our findings across different time horizons. Theoretical properties of time aggregation of univariate GARCH models can for instance be found in Drost and Werker (1996) and Meddahi and Renault (2004).

The paper is organized as follows. In Section 2 we introduce the time-varying specification for copula–GARCH models and its estimation methodology. In Section 3 we describe the data and the computation of the bivariate returns on the Euro and Japanese Yen spot rates, both quoted against the U.S. Dollar. These are deseasonalized bivariate returns at six time horizons from 1 hour to daily. The first step of the copula–GARCH modeling is performed in Section 4. This consists of univariate GARCH filtering and consequent specification tests. In Section 5 we estimate the time-varying dependence paths using several dynamic copula models, including the time-varying copula dynamics introduced by Tse and Tsui (2002) and the time-varying correlation implied by a BEKK model. The evaluation of the estimated time-varying dependence paths against realized correlation and Kendall's tau is given in Section 6. Our results are summarized in Section 7 which concludes the paper.

2. The model specification and estimation

We proceed to the specification of the dynamic copula model starting by introducing the general copula–GARCH type model.

[2] For a survey on multivariate GARCH models see Bauwens et al. (2006).

[3] Another possible alternative, which we opt not to pursue here, is to model regime changes. In the context of time series analysis, see for instance Hamilton (1990).