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International money and stock market contingent claims

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A B S T R A C T

We develop a unified approach with closed-form solutions for pricing bonds, stocks, currencies and their derivatives. The specification assumes a fundamental risk factor represented by a stochastic positive definite matrix following a Wishart autoregressive (WAR) process. By assuming a volatility-in-mean specification for the domestic stock returns and the relative changes of exchange rates, and a domestic stochastic discount factor exponential affine with respect to the fundamental risk, it is possible to derive closed form solutions for the term structures of interest rates and for the risk-neutral probabilities while keeping the flexibility of the model. In particular:

i) The domestic and foreign term structures are jointly affine and correspond to Wishart quadratic term structures, which can ensure the positivity of interest rates;
ii) In this framework where the stock price follows a model with stochastic volatility, we obtain explicit or quasi-explicit formulas for futures and forward contracts, swaps and options. This extends results by Heston (1993) and Ball and Roma (1994).

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1. Introduction

The major part of the financial literature considers separately the problems of bonds, currencies, and stocks pricing. For instance, there exists a large literature on the term structure of interest rates (see e.g., Martellini and Priaulet, 2001; Dai and Singleton, 2003, for surveys). The affine term structure models introduced by Duffie and Kan (1996), (see Duffie, 2008) provide a specification which is easily implementable. Moreover the discrete time version of the affine models allows for lags and switching regime in the dynamics, which greatly improves the fit to bond data (see e.g., Monfort and Pegoraro, 2007). In parallel the models for stock pricing generally assume a constant riskfree rate.[1] and try to reproduce the basic implication of the CAPM, that is the positive relationship between expected return and volatility due to the existence of a risk premium (Merton, 1973).

Coherent models for market indexes and exchange rates (or for term structures and exchange rates) have also been proposed in the literature. For instance, Grabbe, 1983; Amin and Jarrow, 1991; Brace and Musiela, 1997 considered derivative pricing in an international economy from both domestic and foreign perspectives. In particular, closed-form pricing formulas have been derived by Amin and Jarrow (1991, 1993); Brace and Musiela, 1997, for international markets. They are all based on the Gaussian Heath–Jarrow–Morton model (Heath et al., 1992)] Despite the interest of the derived closed-form formulas, these approaches are valid under the restrictive assumption of constant or deterministic volatility.[2] Moreover they do not ensure the positivity of interest rates.

The present paper extends the earlier literature. Its aim is to encompass the affine term structure models and the standard models for stock pricing in a multicountry framework, providing simple closed-form pricing formulas. Compared to Amin and Jarrow (1991), the Wishart quadratic term structure model is taken as a basis, instead of the Heath, Jarrow, Morton's model. The approach still provides closed-form pricing formulas and appears more flexible. In particular i) stochastic volatilities and covolatilities are allowed; ii) stochastic risk premia of CAPM type can be introduced in the return equations; iii) the positivity of interest rates is easy to characterize by means of appropriate parameter restrictions; iv) Last, but not least historical and risk-neutral distributions are modelled coherently.[3] Whereas the risk-neutral analysis is important for derivative pricing, the historical analysis is needed for parameter estimation based on time series observations and for the determination of Values-at-Risk (VaR). In Section 2, we briefly present stylized facts on yield curves and exchange rates that we want to reproduce. Then we introduce a factor model, including idiosyncratic and general factors. The general factors measure a fundamental multivariate risk represented by a stochastic symmetric positive definite matrix. These factors influence i) the stochastic discount factor (sdf) of the domestic country which drives the level of domestic state prices, ii) the stock prices (domestic and foreign market indexes) by means of a stochastic variance-in-mean model, and iii) the relative changes of the exchange rates. By selecting a domestic sdf which is an exponential affine function of the factor and a factor satisfying a Wishart autoregressive (WAR) process (Gourieroux and Sufana, 2006; Gourieroux et al., 2009), we derive affine domestic and foreign term structures in Section 3. The relationship between the domestic and the foreign term structures is illustrated by numerical examples with realistic parameter values. In particular we show the flexibility of the specification in producing various patterns of domestic and foreign term structures. As well-known, it is important to distinguish between forward and futures contracts when the interest rates are stochastic. In an international economy at least one of the domestic and foreign interest rates is stochastic, whenever the exchange rate is not deterministic.[4] Alternative interpretations of the latent factors are discussed in Section 4. Futures and forward contracts written on market indexes or exchange rates are explicitly priced in Section 5. Similarly we consider in Section 6 closed-form pricing formulas for swaps and

[1] Exceptions are Scruggs and Glabadanis, 2003, or Connoly et al., 2005, 2007.

[2] See Section 4 in Miltersen and Schwartz (1998), Assumption 7 in Amin and Jarrow (1991), or Section 2 in Brace and Musiela (1997).

[3] In previous papers (see e.g., Amin and Jarrow, 1991; Miltersen and Schwartz, 1998), the analysis is only done in the risk-neutral world.

[4] Note that the exchange rates are still assumed predictable in (Dungey et al., 2000).