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A theoretical study of premixed turbulent flame development

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Abstract

Flame development in a statistically stationary and uniform, planar, one-dimensional turbulent flow is theoretically studied. A generalized balance equation for the mean combustion progress variable, which includes turbulent diffusion and pressure-driven transport terms, as well as the mean rate of product creation, is introduced and analyzed by invoking the sole assumption of a self-similar flame structure, well-supported by numerous experiments. The assumption offers the opportunity to simplify the problem by splitting the aforementioned partial differential equation into two ordinary differential equations, which separately model spatial variations of the progress variable and time variations of flame speed and thickness. The self-similar profile of the progress variable, obtained in numerous experiments, is theoretically predicted. Closures of the normalized pressure-driven transport term and mean rate of product creation are obtained. The closed balance equation shows that turbulent diffusion dominates during the initial stage of flame development, followed by the transition to counter-gradient transport in a sufficiently developed flame. A criterion of the transition is derived. The transition is promoted by the heat release and pressure-driven transport. Fully developed mean flame brush thickness and speed are shown to decrease when either density ratio or pressure-driven transport increases. Solutions for the development of the thickness are obtained. The development is accelerated by the pressure-driven transport and heat release.

Keywords: Developing premixed turbulent flame; Self-similarity; Burning velocity; Flame thickness

1. Introduction

Although substantial progress has been made in turbulent flame studies over the past decade, the problem of modeling premixed turbulent combustion is still disputed. In the adiabatic and equi-diffusive case, the following equation:

$$\frac{\partial \bar{\rho}\tilde{c}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_j\tilde{c}) = \underbrace{-\frac{\partial}{\partial x_j}(\overline{\rho u_j'' c''})}_{\mathrm{I}} + \underbrace{\bar{\rho}\tilde{W}}_{\mathrm{II}} \qquad (1)$$

is commonly used to simulate practical flames, with other characteristics of reacting mixture (temperature, density, etc.) being calculated for a known combustion progress variable, $\tilde{c}$, by invoking the assumption of thin local fronts [1]. Here, t is the time; x_j and u_j are the coordinates and flow velocity components, respectively; ρ is the gas density; $\tilde{W}$ is the mean rate of product creation; and the Reynolds averages are denoted by overbars, as well as the Favre averages, such as

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$\overline{\rho}\tilde{c} = \overline{\rho c}$, are used, where $c'' = c - \tilde{c}$ and $c' = c - \bar{c}$.

A number of closures of the two terms in the right-hand side (RHS) of Eq. (1) have been developed [1–6] but no model is yet recognized as a well-established predictive tool thoroughly validated in representative, well-defined cases. The general properties of certain models, which do not allow for the so-called pressure-driven (or "counter-gradient") transport (see [1]), have been analyzed [7–11] using the KPP-technique in the sole limit case of a fully-developed flame. For the same limit case, the effects of the pressure-driven transport on flame thickness and speed have been addressed in few papers [1,8,12].

Since typical practical and laboratory premixed turbulent flames are *developing* ones [13], the lack of a general theoretical study of the behavior of a *developing* flame is an essential gap in combustion theory; especially as certain universal features of turbulent flame development are experimentally well-documented, and they challenge many models, as discussed elsewhere [13,14]. The main goal of this research is to theoretically investigate the behavior of developing flames in a statement as general as possible for obtaining an analytical solution. The particular goals are (1) to suggest a generalized balance equation for the mean combustion progress variable, which is sufficiently general to subsume a number of published models relevant to Eq. (1) and is sufficiently simple to be theoretically investigated; and (2) to analyze the solutions of this equation.

2. A generalized balance equation for the mean combustion progress variable

To obtain analytical solutions, we will consider a statistically planar, 1D flame which propagates in a statistically stationary and uniform turbulent mixture, a quite standard problem for theoretical studies of premixed combustion. The following equation:

$$\frac{\partial}{\partial t}(\overline{\rho}\tilde{c}) + \frac{\partial}{\partial x}(\overline{\rho}\tilde{u}\tilde{c}) = \underbrace{\frac{\partial}{\partial x}\left(\overline{\rho}D_t\frac{\partial \tilde{c}}{\partial x}\right)}_{\text{III}} - \underbrace{\rho_u v\frac{\partial f}{\partial x}}_{\text{IV}} + \underbrace{\frac{\rho_u}{\tau_f}w}_{\text{V}}, \tag{2}$$

supplemented with the Bray–Moss model [15]

$$\overline{\rho}\tilde{c} = \frac{\rho_u}{\gamma}\bar{c} = \frac{\rho_u - \overline{\rho}}{\gamma - 1} \tag{3}$$

will be analyzed, where $f(\tilde{c},\gamma) \geqslant 0$ and $w(\tilde{c},\gamma) \geqslant 0$ are arbitrary functions such that $f(\tilde{c}=0) = f(\tilde{c}=1) = w(\tilde{c}=0) = w(\tilde{c}=1) = 0$; $\gamma = \rho_u/\rho_b$ is the density ratio; D_t is the turbulent diffusivity; and $\partial\tilde{c}/\partial x \geqslant 0$. The velocity, v, and time, τ_f, scales are introduced for dimensional reasons.

Term V in the RHS of Eq. (2) is associated with the mean rate of product creation (term II in Eq. (1)) and generalizes a number of known closures (e.g., see [3–5,8]), as discussed elsewhere [14,16]. Models of turbulent flames, which invoke an additional balance equation for flame surface density in order to close term V (see review [2]), should be presented in a more general form, i.e., $w(\tilde{c},\gamma)$ is replaced with $\sigma(x,t,\tilde{c},\gamma)$. However, even these models are subsumed by Eq. (2) as $t \to \infty$, the sole case that has been theoretically studied so far [2,10].

Terms III and IV in Eq. (2) are associated with the transport term I in Eq. (1). Term III is a diffusion closure invoked by many models of turbulent combustion [2,4,7,9–11]. Term IV has been introduced into Eq. (2) to simulate pressure-driven transport, discussed elsewhere [1–3,5,6]. A typical approach to modeling this phenomenon consists of invoking a balance equation for the turbulent flux $\overline{\rho u'' c''}$ [1,3,5,6]. Such models cannot be reduced to Eq. (2) because the function $f(\tilde{c},\gamma)$ in term IV should be replaced with a more general function $f'(x,t,\tilde{c},\gamma)$ for this purpose. However, the models are subsumed by Eq. (2) as $t \to \infty$.

Bray et al. [6] have developed the following simple closure of the transport term:

$$\overline{\rho u'' c''} = \frac{(\gamma - 1)S_L}{2}\overline{\rho}\tilde{c}(1 - \tilde{c}) \tag{4}$$

in the case of a premixed flame in stagnating turbulence (the positive RHS corresponds to the case of $\partial\tilde{c}/\partial x \geqslant 0$). Here, S_L is the laminar flame speed. Term IV in Eq. (2) is a generalization of this closure, but we have also kept the diffusion term III in Eq. (2) for a number of reasons, discussed elsewhere [16,17].

By using the following time, $t_0 = \tau_f$; velocity, $u_0 = (D_t/\tau_f)^{1/2}$; length, $l_0 = (D_t\tau_f)^{1/2}$; and density, ρ_u, scales, Eq. (2) reduces to

$$\frac{\partial}{\partial t'}(\overline{\varrho}\tilde{c}) + \frac{\partial}{\partial z}(\overline{\varrho}\tilde{v}\tilde{c}) = \frac{\partial}{\partial z}\left(\overline{\varrho}\frac{\partial \tilde{c}}{\partial z}\right) - P\frac{\partial f}{\partial z} + w, \tag{5}$$

where $t' = t/t_0$, $z = x/l_0$, $v = u/u_0$, $\varrho = \rho/\rho_u$, and $P = v/u_0$.

This re-normalization of Eq. (2) implies that turbulent characteristics are not changed in the flame, i.e., effects of heat release on the turbulence, discussed elsewhere [1–3,5,6,18], are not studied here.

3. Method of solution

3.1. Self-similarity of premixed turbulent flame structure

Our goal is to obtain solutions to Eqs. (3) and (5) supplemented with the normalized mass balance equation