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Gradient plasticity theory with a variable length scale parameter

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Abstract

The definition and magnitude of the intrinsic length scale are keys to the development of the gradient plasticity theory that incorporates size effects. However, a fixed value of the material length-scale is not always realistic and different problems could require different values. Moreover, a linear coupling between the local and nonlocal terms in the gradient plasticity theory is not always realistic and that different problems could require different couplings. This work addresses the proper modifications required for the full utility of the current gradient plasticity theories in solving the size effect problem. It is shown that the current gradient plasticity theories do not give sound interpretations of the size effects in micro-bending and micro-torsion tests if a definite and fixed length scale parameter is used. A generalized gradient plasticity model with a non-fixed length scale parameter is proposed based on dislocation mechanics. This model assesses the sensitivity of predictions to the way in which the local and nonlocal parts are coupled (or to the way in which the statically stored and geometrically necessary dislocations are coupled). In addition a physically-based relation for the length scale parameter as a function of the course of deformation and the material microstructural features is proposed. The proposed model gives good predictions of the size effect in micro-bending tests of thin films and micro-torsion tests of thin wires.

Keywords: Gradient plasticity; Material length scale; Size effects; Micro-bending; Micro-torsion

1. Introduction

Material length scales or size effects (i.e. the dependence of mechanical response on the structure size) are of great importance to many engineering applications. Moreover, the emerging area of nanotechnology

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exhibits important strength differences that result from continuous modification of the material microstructural characteristics with changing size, whereby the smaller is the size the stronger is the response. There are many experimental observations which indicate that, under certain specific conditions, the specimen size may significantly affect deformation and failure of the engineering materials and a length scale is required for their interpretation. An extensive review of the experimental observations of size effects has been summarized in Abu Al-Rub and Voyiadjis (2004,) and Voyiadjis and Abu Al-Rub (2004). This dependence of mechanical response on size could not be explained by the classical continuum mechanics since no length scale enters the constitutive description. However, the gradient plasticity theory has been successful in addressing the size effect problem. This success stems out from the incorporation of a microstructural length-scale parameter in the governing equations of the deformation description.

Gradient approaches typically retain terms in the constitutive equations with specific order of spatial gradients and coefficients that represent length-scale measures of the deformed microstructure associated with the nonlocal continuum. Aifantis (1984) was one of the first to study the gradient regularization in solid mechanics. An extensive review of the recent developments in gradient theories can be found in Voyiadjis et al. (2003, 2004) and Abu Al-Rub and Voyiadjis (2004,). A short review of the developments of the gradient-dependent theory is presented here. Other researchers have contributed substantially to the gradient approach with emphasis on numerical aspects of the theory and its implementation in finite element codes: Lasry and Belytschko (1988), Zbib and Aifantis (1988), and de Borst and co-workers (e.g. de Borst and Mühlhaus, 1992; de Borst et al., 1993; Pamin, 1994; de Borst and Pamin, 1996). Gradient thermodynamic damage models were also introduced by Fremond and Nedjar (1996) and Voyiadjis et al. (2001, 2003, 2004).

In parallel, other approaches that have length-scale parameters in their constitutive structure (commonly referred to as nonlocal theories) have appeared as an outgrowth of earlier work by Eringen (e.g. Eringen and Edelen, 1972) and Bazant (e.g. Pijaudier-Cabot and Bazant, 1987; Bazant and Pijaudier-Cabot, 1988). Nonlocal models also abandon the assumption that the stress at a given point is uniquely determined by the history of strain and temperature at this point only. They take into account possible interactions with other material points in the vicinity of that point. Theoretically, the stress at a point can depend on the strain history in the entire body, but the long-range interactions certainly diminish with increasing distance, and can be neglected when the distance exceeds the length of interaction. Early studies on nonlocal elasticity, motivated by homogenization of the atomic theory of Bravais lattices, aimed at a better description of phenomena taking place in crystals on a scale comparable to the range of interatomic forces. They showed that nonlocal continuum models can approximate the dispersion of short elastic waves and improve the description of interactions between crystal defects such as vacancies, interstitial atoms, and dislocations. Eringen (e.g. Eringen and Edelen, 1972) introduced the idea of nonlocal continuum in elasticity and phenomenological hardening plasticity; while, Bazant (e.g. Pijaudier-Cabot and Bazant, 1987; Bazant and Pijaudier-Cabot, 1988) extended the nonlocal concept to strain softening materials and introduced the nonlocal damage theory.

Another class of gradient theories have advocated in the last decade that the stress tensor of the resulting three-dimensional constitutive equations is an *asymmetric stress tensor*. These theories assume higher-order gradients of the displacement field (e.g. Fleck et al., 1994; Fleck and Hutchinson, 1993, 1997, 2001; Nix and Gao, 1998; Gao et al., 1999a,b; Huang et al., 2000a; Gao and Huang, 2001; Hwang et al., 2002). This group of theories is in fact a particular case of generalized continua, such as *micromorphic continua* (Eringen, 1968), or *continua with microstructure* (Mindlin, 1964), which were all inspired by the pioneering work of the Cosserat brothers (Cosserat and Cosserat, 1909). The *Cosserat continuum* (or *micropolar continuum*) enhances the kinematic description of deformation by an additional field of local rotations, which can depend on the rotations corresponding to the displacement field, i.e. on the skew-symmetric part of the displacement gradient for the small displacement theory, or on the rotational part of the polar decomposition in the large-displacement theory. In this connection, a similarly motivated strain gradient theory of plasticity