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Multiple spatial and temporal scales method for numerical simulation of non-classical heat conduction problems: one dimensional case

Hongwu Zhang *, Sheng Zhang, Xu Guo, Jinying Bi

State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China

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Abstract

A multiple spatial and temporal scales method is studied to simulate numerically the phenomenon of non-Fourier heat conduction in periodic heterogeneous materials. The model developed is based on the higher-order homogenization theory with multiple spatial and temporal scales in one dimensional case. The amplified spatial scale and the reduced temporal scale are introduced respectively to account for the fluctuations of non-Fourier heat conduction due to material heterogeneity and non-local effect of the homogenized solution. By separating the governing equations into various scales, the different orders of homogenized non-Fourier heat conduction equations are obtained. The reduced time dependence is thus eliminated and the fourth-order governing differential equations are derived. To avoid the necessity of C^1 continuous finite element implementation, a C^0 continuous mixed finite element approximation scheme is put forward. Numerical results are shown to demonstrate the efficiency and validity of the proposed method.

Keywords: Non-Fourier heat conduction; Multiple scale method; Homogenization; Non-local model

1. Introduction

The classical Fourier law is well known and has been used successfully for analysis of steady heat conduction process under long time heating and unsteady process with quick propagation speed of the thermal wave. However, Fourier law breaks down in situations involving very short times, high heat fluxes, and at

* Corresponding author. Tel.: +86 41184706249; fax: +86 41184708769.
E-mail address: zhanghw@dlut.edu.cn (H.W. Zhang).

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very cryogenic temperatures (Baumeister and Hamill, 1969). The anomaly of this classical theory is from the assumption that the heat flux vector and the temperature gradient across a material volume occur at the same instant of time. Such an immediate response results in an infinite speed of heat propagation.

The heat sources such as laser and microwave with very high frequency and extremely short duration have been used widely in modern technology in past years. This leads to the increase of research interest of non-Fourier law. The mathematical description of non-Fourier heat conduction law, which represents the time lag of heat waves, is a hyperbolic type differential equation. As has been pointed out by many researchers, this non-classical heat conduction law has its great value in many practical applications, such as laser penetration and welding, explosive bonding, electrical discharge machining, heating and cooling of microelectronic elements involving a duration time of nanoseconds or even picoseconds in which the energy is absorbed within a distance of microns from the surface. In order to associate a finite heat propagation speed, Cattaneo (1958) and Vernotte (1961) modified Fourier law by including a relaxation model. Non-Fourier heat conduction in solids with different shapes and boundary conditions has been studied extensively. Frankel et al. (1987), using flux formulation of hyperbolic heat conduction equation, gave an analytical solution for a finite slab under boundary condition of rectangular heat pulse. Ozisik and Tzou (1994) analyzed the special features in thermal wave propagation, and the thermal wave model in relation to the microscopic two-step model. Kaminski (1990) determined experimentally the values of a relaxation time for non-homogeneous inner structure materials. Tzou (1995) presented a universal constitutive equation between the heat flux vector and the temperature gradient. Jiaung et al. (2003) studied the effect of the phase lag of temperature gradient. On the other hand, the stochastic finite element method was successfully applied in displacement-based finite element method in transient heat transfer for heterogeneous media, which is based on the second order perturbation second central probabilistic moment method (Hien and Kleiber, 1997; Kamiński and Hien, 1999a,b).

It has been found that the multi-scale asymptotic homogenization approach is wide acceptance for the study of heterogeneous structures due to its systematic mathematical approach and ability to account for multi-scale features (Bakhvalov and Panasenko, 1989; Benssousan et al., 1978; Chung et al., 2004; Sanchez-Palencia, 1980). The mathematical homogenization method was used as an alternative approach to compute effective constitutive parameters of complex materials with a periodic structure in Hassani and Hinton (1998). To capture the effects of microstructural changes on the overall response of a composite made of bodies in elastic and elastic–plastic contact, numerical homogenized constitutive law is then defined in Zhang et al. (1999) and Zhang and Schrefler (2000) for the global behavior of the heterogeneous materials. For the composite with detailed information on the microgeometry, Kamiński (2000) extended the effective modules method by using the finite element method or the boundary element method in numerical implementations, which enable direct computations of the effective characteristics. Gambin and Kroner (1989), and Boutin (1996) have studied the role of higher-order terms in the asymptotic expansion in statics. Boutin and Auriault (1993) demonstrated the terms of a higher-order successively introduced effects of dispersion and attenuation in elastokinetics. A single-frequency time-dependence is assumed prior to the homogenization process (Kevorkian and Bosley, 1998). Chen and Fish (2001) and Fish and Chen (2001a,b) investigated the problem of secularity introduced by the higher order multiple spatial–temporal scale approximation of the initial boundary value problem in periodic heterogeneous media. Fish et al. (2002) developed a non-local approach independent of the slow time scale considered the problem of secularity.

Recently, considerable interest has been generated toward transient heat transfer by the multi-scale asymptotic homogenization method and its potential applications in engineering and technology. Boutin (1995) studied the heat propagation in media with a periodic microstructure. It is shown that the higher terms introduce successive gradients of temperature and tensors, characteristic of the microstucture, which result in non-local effects. A systematic way of obtaining the effective viscoelastic moduli in time and frequency domain is presented for periodic microstructures in Yi et al. (1998), the effective modulus is formulated using the asymptotic homogenization method. Yu and Fish (2002) developed a systematic approach