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Classification of compatibility paths of SDOF mechanisms

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Abstract

The compatibility paths of mechanisms with a single degree-of-freedom typically form sets of curves in the global representation space. We classify the different cases of compatibility by introducing an energy function. The result obtained also depends on which element of the mechanism is regarded as driven. The different singularity types are demonstrated by examples (split-vanish point, limit point, asymmetric bifurcation, infinitely degenerate bifurcation, hilltop point, compatibility surface).

Keywords: Mechanism; Compatibility path; Energy function; Singularity; Bifurcation; Classification

1. Introduction

A mechanism consisting of rigid bars with prescribed length and a given topology is called a mechanism with a single degree-of-freedom if it typically has compatible positions where applying a suitable displacement to a suitable element, the displacement of the other elements can be uniquely determined. Compatibility paths form a set of points which belong to compatible positions in the space of the state variables chosen to define the position of the mechanism. The compatibility paths usually consist of lines, which can intersect one another (bifurcation points).

Bifurcation points arise from special geometric configurations. If a mechanism is created with a special geometry, it may have certain positions where the number of instantaneous kinematic degrees-of-freedom increases. At these points the mechanism can change shape and continue its motion along a different path.

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Bifurcations of compatibility paths have been studied by several researchers. Tarnai (1999) and Litvin (1980) have shown mechanisms producing asymmetric bifurcations. Lengyel and You (2003) discussed further examples and paralleled this phenomenon to the well-known equilibrium bifurcations of elastic structures. They made further examination with the aid of the elementary catastrophe theory (Lengyel and You, 2004). Their method was based on the analogy between equilibrium and compatibility equations.

The aim of this paper is to classify the points of the compatibility paths, especially bifurcation points. The classification utilizes, where possible, the classification system developed for the equilibrium paths of elastic structures with a single loading parameter. Knowing the equilibrium paths of the perfect structure, the approximate calculation of the diagram of the paths associated with small imperfections becomes considerably easier. The equilibrium analysis employs the total potential energy function of the structure. Similar functions can be formulated for mechanisms as well, such as the ones proposed by Tarnai (1990) or Géradin (2001).

In this paper we introduce an energy function, the minimum points of which define the points of the compatibility paths, and which helps to identify sections of the compatibility paths which can divide or disappear due to suitable imperfections. Such points have been called 'split-vanish' points by Lengyel (2002), which name we adopt here.

2. Choosing the state variables

The coordinates of all nodes of the structure uniquely determine the position of the structure, though other variables are usually used in order to minimize the number of state variables. It is to be mentioned that in many cases the displacement of an element can be uniquely defined by angles in larger intervals than by the Cartesian coordinates of a chosen point. One may think that one state variable is sufficient to describe the position of a SDOF mechanism, but unfortunately it is not always sufficient for the global description of the compatibility paths. The space of the variables required for a unique description is called global representation space (GRS) (Gáspár et al., 1997). If two compatibility paths intersect in this space, then it indeed belongs to a bifurcation point of the compatibility paths.

Fig. 1 shows three mechanisms with a single degree-of-freedom to demonstrate the required number of the state variables and a practical way of choosing them. Fig. 1a shows a single bar the position of which is uniquely given by angle α. As all points in this one-dimensional space belong to compatible positions, such simple structures are not considered in the following.

In Fig. 1b angle α uniquely defines node A, the general position of which is associated with two different compatible positions of the other bar. Another datum (i.e. y) is required to distinguish between the two positions denoted by continuous and dashed lines, respectively.

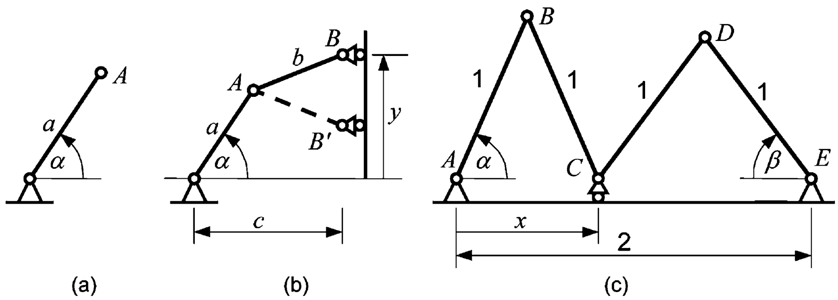

Fig. 1. SDOF mechanisms.