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A spectral super element for modelling of plate vibration. Part 1: general theory

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Abstract

The dynamic response of vibrating structures is studied with a proposed merger of the standard finite element method with the more computationally efficient spectral finite element method. First a plate structure is modelled with a newly developed spectral super element. Then this element is coupled to other parts that can have a more complex geometry and are modelled entirely with conventional finite elements. Some numerical examples are given to illustrate and validate the developed method and studies of numerical stability are also presented. In an accompanying paper the predicted and measured response of a turbulence excited aircraft panel are compared.

1. Introduction

The finite element method (FEM) provides a mathematically stable environment to simulate dynamic response and it also allows complex geometrical structures to be modelled. However, with high-frequency excitation and distributed random excitation, many structures of interest require impossibly large computer models.

A number of methods to reduce the number of degrees of freedom (dof) and increase the computational efficacy have been presented in the past, for example the spectral finite element

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method (SFEM) [1], the transfer matrix method [2], the dynamic stiffness method (DSM) [3,4], the wave-based method [5] and scale-independent elements [6]. These are all formulated in the frequency domain and the frequency-dependent formulation simplifies the inclusion of frequency-dependent material characteristics and boundary condition. The basis functions in these methods are exact or approximate solutions to the local equations of harmonic motion and the elements are assembled as in the standard FEM.

For sub structures that have uniform properties along one direction, say, the x-direction, the local solution on the cross section, i.e. in the yz plane, can conveniently be approximated by polynomial displacement functions. This two-dimensional finite element technique first appeared in Ref. [7] and more recently in references describing wave propagation in laminated composite structures [8,9], thin-walled beams [10,11], rib-stiffened panels [12], anisotropic shells and beams [13,14], fluid-filled pipes [15,16], pre-stressed and curved shells [17] and a wind tunnel [11]. This finite element technique will be referred to here as the waveguide FEM and one of its advantages compared to conventional FEM is that different wave types are readily identified and can be analysed, allowing for a physical understanding of the investigated structure.

This study proposes a new application of the waveguide FEM, where the displacement is described as a combination of found wave solutions. This displacement is then expressed as a function of the nodal displacement at the waveguide ends and inserted in the equation of virtual work. Requiring the first variation of these displacements to be zero, the structural response is found. This method can be seen as a merger of the waveguide FEM with the SFEM and was named spectral super element method (SSEM).

These spectral super elements are characterized by the ease with which they can be put into an assembly with proper coupling to neighbouring elements. Hence, large wave-carrying parts of a structure can be described by this method, whereas smaller parts with complex geometry are modelled entirely by finite elements. In Ref. [3], a super element was created by condensing all the interior dof for a finite element method. Then, the remaining dof were reduced in order to be compatible with the waveguide dof at the joints. With the method presented here such a reduction is not necessary as long as the same nodes are used at the joints with the FEM and SSEM.

A somewhat related finite strip method (FSM) [18,19] also uses polynomial displacement functions to represent the displacement of the cross-section for thin flat-walled structures. This FSM now exists in a number of variants and has been successfully applied to composite laminated structures, e.g. Ref. [20]. However, since the displacement in the direction of the waveguide is described either as a combination of beam eigenfunctions or polynomial spline functions, the FSM leads to a different analysis than with the SSEM. Nevertheless, many application areas for the FSM are most likely also worth studying with the SSEM.

The following is a general theory for a spectral super element. In an accompanying paper [21] distributed excitation is investigated and the element is used to predict the turbulent boundary layer (TBL) response of a clamped plate. Much effort was invested in the validation of the element against similar elements described by either the DSM or the FEM. Two examples of the coupling of spectral super elements to finite elements are provided.