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Speed-flow relations and cost functions for congested traffic Theory and empirical analysis

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Abstract

A dynamic 'car-following' extension of the conventional economic model of traffic congestion is presented, which predicts the average cost function for trips in stationary states to be significantly different from the conventional average cost function derived from the speed-flow function. When applied to a homogeneous road, the model reproduces the same stationary state equilibria as the conventional model, including the hypercongested ones. However, stability analysis shows that the latter are dynamically unstable. The average cost function for stationary state traffic coincides with the conventional function for non-hypercongested traffic, but rises vertically at the road's capacity due to queuing, instead of bending backwards. When extending the model to include an upstream road segment, it predicts that such queuing will occur under hypercongested conditions, while the general shape of the average cost function for full trips does not change, implying that hypercongestion will not occur on the downstream road segment. These qualitative predictions are verified empirically using traffic data from a Dutch bottleneck. Finally, it is shown that reduced-form average cost functions, that relate the sum of average travel cost and average schedule delay costs to the number of users in a dynamic equilibrium, certainly need not have the intuitive convex shape, but may very well be concave – despite the fact that the underlying speed-flow function may be convex.

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1. Introduction

Notwithstanding the long history of the economists' and engineers' study of road traffic congestion and congestion pricing (see Pigou, 1920; Knight, 1924; Wardrop, 1952; Walters, 1961; Vickrey, 1969), academia has not yet reached general consensus on the fundamentals that should underlie such analysis. This is illustrated by the relatively large number of comments and replies that papers on congestion modelling seem to trigger (Else, 1981, 1982; versus Nash, 1982; De Meza and Gould, 1987; versus Alan Evans, 1992a; Andrew Evans, 1992b, 1993; versus Hills, 1993; and Ohta, 2001a,b; versus Verhoef, 2001b). Much of this debate concerns the derivation of cost functions for road use for the conventional static economic model of traffic congestion—which uses the speed-flow function to derive the average cost function (see also Section 2 below)—and on the interpretation of the 'hypercongested' segment (see below) of the resulting backward-bending cost functions.

This paper reconsiders the derivation of cost functions from speed-flow relations from a dynamic perspective, with an emphasis on the phenomenon of 'hypercongestion'. The paper first summarizes the main results obtained with a dynamic extension of the conventional model, presented in two earlier papers (Verhoef, 2001a, 2003), and will then test these empirically by contrasting the model's qualitative, aggregate predictions to insights obtained from empirical data for a well-known bottleneck on the Dutch road network (The Coenplein, near Amsterdam). The proposed dynamic extension involves a simple first-order car-following model, linking a driver's instantaneous speed choice to the distance from her leader. Because distance is the inverse of density, this car-following model is easily made consistent with the conventional model for stationary state traffic. Two variants of this model will be considered: one in which there is a single homogeneous road of constant capacity, and one in which there is a bottleneck halfway this road due to a reduction in the number of lanes.

Dynamic stability analysis for the homogeneous road setting demonstrates that hypercongested stationary states are in fact dynamically unstable. This means that there are no equilibrium paths towards such a stationary state from any other stationary state (hypercongested or not). This finding will be substantiated in Section 3, and the implications for the shape of the average cost function (for full trips) are identified. These are that this function coincides with the conventional function for normally congested speeds, but instead of bending backwards it rises vertically at the road's capacity (provided queuing at its entrance is allowed).

In contrast, when applied for a road with a bottleneck, the model shows that in a dynamic equilibrium with endogenous departure times (á *la* Vickrey, 1969), hypercongestion will occur with certainty during the equilibrium peak, provided the equilibrium number of users over the peak, relative to the capacity of the downstream road segment, is sufficiently large. In particular, hypercongestion will prevail on the upstream, high-capacity segment of the road, whereas the speed on the downstream, low-capacity segment will asymptotically approach the value consistent with the maximum