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Rough operations on Boolean algebras

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Abstract

In this paper, we introduce two pairs of rough operations on Boolean algebras. First we define a pair of rough approximations based on a partition of the *unity* of a Boolean algebra. We then propose a pair of generalized rough approximations on Boolean algebras after defining a basic assignment function between two different Boolean algebras. Finally, some discussions on the relationship between rough operations and some uncertainty measures are given to provide a better understanding of both rough operations and uncertainty measures on Boolean algebras.

Keywords: Boolean algebra; Rough operations; Uncertainty measures

1. Introduction

Rough set theory was introduced by Pawlak [16] to generalize the classical set theory. In rough set theory, given an equivalence relation on a universe, we

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can define a pair of rough approximations which provide a lower bound and an upper bound for each subset of the universe. Rough approximations can also be defined equivalently by a partition of the universe which is corresponding to the equivalence relation [11]. In [14,15], Yao generalized rough set theory by generalizing the equivalence relation to a more general relation, and then interpreted belief functions using the generalized rough set theory in [12].

In [6,7], Bayesian theory and Dempster-Shafer theory [3] are extended to be constructed on Boolean algebras. This provides a more general framework to deal with uncertainty reasoning. In this paper, we define a pair of dual rough set operations on Boolean algebras to interpret belief functions and some other uncertainty measures on Boolean algebras. The new pair of rough operations coincide with $\cap$- and $\cup$-homomorphisms in *interval structure* [9,10]. The difference is that the $\cap$- and $\cup$-homomorphisms are defined by some axioms, whilst the (generalized) rough operations are defined by a (generalized) partition. The lower approximation defined in this paper can be used to interpret belief functions on Boolean algebras in [7]. We establish a one to one correspondence between belief functions and pairs consisting of a Bayesian function and a lower approximation given by a basic assignment. This result is more general than the corresponding relation between rough sets and belief functions given in [12].

This paper is organized as follows. In the first two sections, we introduce some important concepts of Boolean algebra and evidential measures on Boolean algebras. In Section 4, we first define a pair of rough approximations based on a partition of the unity of a Boolean algebra, then propose a pair of generalized rough approximations after defining a basic assignment function between two different Boolean algebras. In Section 5, we discuss relationship between rough operations and some uncertainty measures. Finally, we summarize the paper in Section 6.

2. A brief review of Boolean algebra

A Boolean algebra [4] is a 6-tuple $\langle \chi, \cup, \cap, ', \Phi, \Psi \rangle$ where χ is a set: $\Phi, \Psi \in \chi$, and for every $A, B \in \chi$ there exist $A \cup B \in \chi$, $A \cap B \in \chi$ satisfying the following conditions

(b1) commutative laws: $A \cup B = B \cup A$, $A \cap B = B \cap A$;
(b2) associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$;
(b3) there exist $\Phi, \Psi \in \chi$: $A \cup \Phi = A$, $A \cap \Psi = A$ and $\Phi \neq \Psi$;
(b4) for every $A \in \chi$ there exists an $A' \in \chi$ such that $A \cup A' = \Psi$, $A \cap A' = \Phi$;
(b5) distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \; A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$