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Hierarchical TS fuzzy system and its universal approximation

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Abstract

An efficient tool to deal with the 'rule explosion' problem is the hierarchical system by which a fuzzy system can be decomposed into a number of hierarchically connected low-dimensional systems. In this paper a generalized hierarchical Tagaki–Sugeno (TS) system is built. It is shown that the input–output (I/O) relationship of this generalized hierarchical system can be represented as one of a standard TS fuzzy system. And the system approximation capability is analyzed by taking piecewise linear functions as a bridge. By constructive method it is proven that the hierarchical fuzzy systems (HFS's) can be universal approximators. For the given approximation accuracy, an estimation formula about the number of the rules needed in the HFS is established. Finally some simulation examples confirm that the HFS's with smaller size rule base can approximate the given functions with high accuracy. The results obtained here provide us with the theoretical basis for various applications of HFS's.

Keywords: TS fuzzy system; Hierarchical system; Universal approximator; Piecewise linear function

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1. Introduction

Fuzzy systems have emerged as one of the most active and fruitful research fields [17]. In many real areas, such as pattern recognition [17], automatic control [1,17], system identification [16] etc., we can find many successful applications of fuzzy systems. Also the research on fuzzy systems is of theoretic importance [17,20,23,24]. For instance, through studying approximation capability of fuzzy systems systematically we can build the approximation theory of systems [23,24].

In most rule based fuzzy systems, the fuzzy rule base consisting of a number of inference rules defined as 'IF…THEN…' is a key part. In the paper the fuzzy rule base is assumed to be complete, that is, the rule base is valid for all possible conditions. As the number of the system input variables increases the number of rules in the complete fuzzy rule base increases exponentially. That is the 'Rule explosion' problem, which is in nature the 'curse of dimensionality' which exists in many fields [6,7]. That will not only generate the complicated system structures, but also cause long computational time, even memory overload of the computer.

To make fuzzy systems usable in real complex systems, we have to deal with the 'rule explosion' problem. A few of methods for the problem have so far been put forward [2,3,6,7,13]. In the fuzzy systems or fuzzy controllers, two classes of such methods are significant. One is based on the equivalence of 'intersection rule configuration' and 'union rule configuration' [2–4,13]. That is, if P and Q are two antecedents, and R is a consequent, then

$$[(P \wedge Q) \Rightarrow R] \Longleftrightarrow [(P \Rightarrow R) \vee (Q \Rightarrow R)].$$

Another one is to introduce a hierarchical system configuration [8,14,15,18,19], i.e. instead of applying a fuzzy system with higher-dimensional input, a number of lower-dimensional fuzzy systems are linked in a hierarchical fashion. By such a hierarchy, the number of the fuzzy rules will increase linearly with the number of the input variables. So the HFS's can be efficiently used in some large scale systems. And the application fields of fuzzy systems are undoubtedly extended and expanded. The paper focuses on the HFS's mainly in input–output (I/O) relationship representation and approximating properties.

Naturally we may put forward an important problem, that is, how can the approximating capability of a HFS be analyzed? Kikuchi et al. in [8] show that it is impossible to utilize a HFS to build the precise expression of an arbitrarily given function. So we have to analyze the approximate representation of a function by the HFS's. Whether can the HFS's be universal approximators or not? If a function is continuously differentiable on the whole space, Wang in [18] shows the arbitrarily close approximation of a function by the HFS's; and he also in [19] gives the sensitivity properties of HFS's and designs a suitable system structure. For each compact set U and an arbitrarily given