Journal of Mathematical Economics 61 (2015) 67–73

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

New conditions for the existence of Radner equilibrium with infinitely many states[☆]

Bogdan Klishchuk

Research School of Economics, The Australian National University, ACT 2601, Australia

ARTICLE INFO

Article history:
Received 17 November 2014
Received in revised form
11 June 2015
Accepted 6 August 2015
Available online 14 August 2015

Keywords:
Differential information
Asymmetric information
Independent information
Common knowledge
No-trade theorems
Competitive equilibrium

ABSTRACT

In atomless differential information economies, equilibria are known not to exist prevalently even when agents are risk averse expected utility maximizers. The notion of prevalence involves essentially picking an economy at random. In this paper, however, we establish existence results with economically meaningful assumptions on the information structure. We obtain existence when agents have independent information, and also when the total endowment of the economy is common knowledge.

1. Introduction

In 1968 Radner explored how far one could go in applying the theory of competitive equilibrium to the case of differentially informed agents. He tailored a way to model information asymmetries so that the standard notion of Walrasian equilibrium would apply. His economic model has subsequently become known as a differential information economy, and its Walrasian equilibrium is commonly referred to as Radner equilibrium (see Section 2). Radner concluded that standard theorems on the existence of Walrasian equilibrium continued to hold, but that referred only to the case of finitely many states of nature and finitely many state-independent commodities (available for consumption in each state), as infinite-dimensional equilibrium theory was only in its infancy back then.

It is now understood that the situation with infinite-dimensional commodity spaces is more subtle. Podczeck and Yannelis (2008) established the existence of Radner equilibrium with infinitely many state-independent commodities, but the case of infinitely many states of nature was left behind. Tourky and Yannelis (2003) and Podczeck et al. (2008) have discovered prevalent non-existence conditions peculiar to the case of infinitely many states even with preferences confined to risk averse expected utility. They utilize the notion of prevalence introduced by Anderson and Zame (2001). The basic idea behind this result is that in atomless differential information economies agents with different priors and information seek to specialize their optimal consumption on a null event, and the duality condition characterizing the existence of equilibrium derived in Aliprantis et al. (2004, 2005) cannot hold.

Radner equilibrium with infinitely many states is nevertheless known to exist under restrictive yet economically meaningful assumptions on the information structure. Such an assumption was found by Hervés-Beloso et al. (2009). They suppose that each agent observes a public and a private signal; the public signal may take infinitely many values, but private signals are restricted to take only finitely many values. We introduce two new economically meaningful conditions that guarantee the existence of Radner equilibrium with infinitely many states.

Our first condition (Section 3) requires that agents' information σ-algebras (or signals) are independent. If, in addition, there is only one commodity available for consumption in each state, then there exists a unique Radner equilibrium in which there is no trade. With more commodities per state, however, agents might be willing to trade, and the problem of existence becomes more challenging. In this more general scenario, we also make an assumption that limits the substitutability of one state-independent commodity by others.

Our second condition (Section 4) requires that the total endowment of the economy is common knowledge. This is the same as

[☆] This research was inspired by two working papers (Tourky and Yannelis, 2003; Podczeck et al., 2008) kindly provided to the author by his Ph.D. supervisor at the ANU, Professor Rabee Tourky.

E-mail address: bogdan.klishchuk@anu.edu.au.

http://dx.doi.org/10.1016/j.jmateco.2015.08.003
0304-4068/

saying that the total endowment belongs to every agent's informationally constrained consumption set. We also make somewhat unusual assumptions on preferences, but we show that they are implied by standard assumptions if agents exhibit a degree of risk aversion. In particular, this risk aversion is satisfied in the standard case of expected utility with concave Bernoulli functions. We also give an example of non-expected utility preferences satisfying all our assumptions.

Let us briefly mention the importance of infinite state spaces. They arise naturally, for instance, if uncertainty is resolved sequentially over an infinite horizon (see Shreve, 2004). In addition, they are often utilized for the sake of mathematical convenience. For example, if one wants to work with continuously distributed random variables (say, agents' signals about the return of a risky asset), then the state space must be infinite. These considerations originally motivated Bewley (1972) and other authors to introduce infinite-dimensional commodity spaces into general equilibrium theory.

The existence of Radner equilibrium is an active area of research. Recent contributions include Xanthos (2014) and Yoo (2013).

2. Model

We have a finite set of *agents* $I = \{1, \dots, m\}$ who face exogenous *uncertainty* described by a probability space $(S, \mathscr{F}, \mu)$. There is a finite nonempty set C of *state-independent commodities* available for consumption in each state. Agents are differentially informed, which simply means that they have heterogeneous ability to discern events in $\mathscr{F}$. Agent i can only discern events that belong to some sub-σ-algebra $\mathscr{F}_i$ of $\mathscr{F}$. If $\mathscr{G}$ is any sub-σ-algebra of $\mathscr{F}$, we denote the space $(L_1(S, \mathscr{G}, \mu|_{\mathscr{G}}))^C$ by $L_1(\mathscr{G})$ and its positive cone by $L_1^+(\mathscr{G})$. Notice that we can identify points $x \in L_1(\mathscr{G})$ and $y \in L_1\left(S, \mathscr{G}, \mu|_{\mathscr{G}}, \mathbb{R}^C\right)$ satisfying $x_c(s) = (y(s))_c$ for all $s \in S$ and $c \in C$. We take $L_1(\mathscr{F})$ as our *commodity space* and let $L_1^+(\mathscr{F}_i)$ be the informationally constrained *consumption set* of agent i. The agent has a *preference correspondence* $P_i : L_1^+(\mathscr{F}_i) \twoheadrightarrow L_1^+(\mathscr{F}_i)$ and an *initial endowment* $\omega_i \in L_1^+(\mathscr{F}_i)$. We let $\omega = \sum_{i=1}^m \omega_i$ denote the total initial endowment.

An *allocation* is a vector $x \in \Pi_{i=1}^m L_1^+(\mathscr{F}_i)$ such that $\sum_{i=1}^m x_i = \omega$. A *price system* is an element of $(L_\infty(S, \mathscr{F}, \mu))^C$, the topological dual of $L_1(\mathscr{F})$. Given a price system p, the value of a commodity bundle $x \in L_1(\mathscr{F})$ is simply $p \cdot x = \sum_{c\in C} E(p_c x_c)$. An allocation x is said to be a *Radner equilibrium* if there exists a price system p such that, for all $i \in I$, we have $p \cdot x_i \le p \cdot \omega_i$, and $y \in P_i(x_i)$ implies $p \cdot y > p \cdot \omega_i$.

A remark about the interpretation of the probability measure μ is in order now. Technically, our model would not change if we replaced μ by another measure ν as soon as μ-null sets coincided with ν-null sets. This is because spaces of (equivalence classes of) μ-integrable and ν-integrable random variables are lattice isometric. But concepts of independence and risk aversion, which are of fundamental importance to the theory of choice under uncertainty and are of use in this paper, are not immune to such a change of measures. If two random variables are independent with respect to μ, they are not necessarily independent with respect to ν. If a preference relation is risk averse with respect to μ, it is not necessarily risk averse with respect to ν. To be able to interpret mathematical independence as a reflection of causal independence in the real world, we must suppose that μ is a "true" probability measure. On the other hand, the assumption of risk aversion with respect to μ is hard to justify unless μ is supposed to be our agents' common belief. So we can view μ as a "true" probability measure in Section 3, where we make use of the independence assumption, and as our agents' common belief in Section 4, where risk aversion comes into play.

We use expected values and conditional expectations extensively in the exposition of our results. If $x \in L_1(\mathscr{F})$ and $\mathscr{G}$ is a sub-σ-algebra of $\mathscr{F}$, the symbol $E(x)$ denotes the vector $a \in \mathbb{R}^C$ in which $a_c = E(x_c)$ for all $c \in C$, while the notation $E(x|\mathscr{G})$ stands for the element $y \in L_1(\mathscr{G})$ in which $y_c = E(x_c|\mathscr{G})$ for all $c \in C$.

We conclude this section by listing below standard assumptions that would typically be required for an existence proof even without information asymmetries, i.e. when $\mathscr{F}_i = \mathscr{F}_j$ for all $i, j \in I$, as in Section 9.1 of Aliprantis et al. (2001). Assumption (A6) is a version of the properness condition introduced by Tourky (1998). We will refer to these assumptions later.

(A) The following is true for every $i \in I$ and some $v \in \Pi_{j=1}^m L_1\left(\mathscr{F}_j\right)$ satisfying $\sum_{j=1}^m v_j \le \omega$ and $v_j > 0$ for all $j \in I$.
- (1) P_i is *irreflexive*, i.e. $x \notin P_i(x)$ for all $x \in L_1^+(\mathscr{F}_i)$.
- (2) P_i is *convex-valued*, i.e. $P_i(x)$ is a convex set for all $x \in L_1^+(\mathscr{F}_i)$.
- (3) P_i is *strictly monotone*, i.e. $x \in L_1^+(\mathscr{F}_i)$ implies $x + y \in P_i(x)$ for all $y \in L_1^+(\mathscr{F}_i) \setminus \{0\}$.
- (4) P_i has *open values*, i.e. $P_i(x)$ is open in $L_1^+(\mathscr{F}_i)$, relative to a linear topology on $L_1(\mathscr{F})$, for all $x \in L_1^+(\mathscr{F}_i)$.
- (5) P_i has *weakly open lower sections*, i.e. for every $z \in L_1^+(\mathscr{F}_i)$ the set $P_i^{-1}(z) = \left\{y \in L_1^+(\mathscr{F}_i) : z \in P_i(y)\right\}$ is weakly open in $L_1^+(\mathscr{F}_i)$.
- (6) P_i is *proper* in the sense that there exists a convex-valued correspondence $\hat{P}_i : L_1^+(\mathscr{F}_i) \twoheadrightarrow L_1(\mathscr{F})$ such that for each $x \in L_1^+(\mathscr{F}_i)$
  - (i) $x + v_i$ is an interior point of $\hat{P}_i(x)$ and
  - (ii) $\hat{P}_i(x) \cap L_1^+(\mathscr{F}_i) = P_i(x)$.

3. Independent information

In this section we establish the existence of Radner equilibrium when agents' information σ-algebras are independent. We consider the case of a single commodity per state separately first. In this scenario we obtain the existence of a unique Radner equilibrium in which there is no trade. This result is in accord with Koutsougeras and Yannelis (1993), who prove that only the initial allocation belongs to the private core when agents have independent information.[1] However, our main contribution is to show that the initial allocation can actually be supported by some price system. We cannot resort to the second welfare theorem or the converse part of the core equivalence theorem to obtain supporting prices. Tourky and Yannelis (2003) and Podczeck et al. (2008) demonstrate that these theorems do not hold for differential information economies with infinitely many states.

With a single commodity per state, we require only pairwise independence of agents' information. This assumption is expressed formally in (B) below.

Assumption (C), with a single commodity per state and monotonicity (A3), is satisfied whenever in the single-agent economy corresponding to each agent i we can find a Walrasian equilibrium. Technically, this assumption requires the strict separation of the sets $\{\omega_i\}$ and $P_i(\omega_i)$, which need not be convex, by a normalized continuous linear functional. This is implied by a wide variety of conditions on preferences. One set of sufficient conditions for Assumption (C) is given by Assumption (A) with v_i replaced by ω_i (Aliprantis et al., 2001, Corollary 9.2).

Assumption (D) is used in the proof of uniqueness only. This monotonicity condition is implied by strict monotonicity as stated in (A3).

[1] The author thanks Professor Nicholas Yannelis for bringing this result to his attention.