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Centralized allocation in multiple markets[☆,☆☆]

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ABSTRACT

We provide a characterization result for the problem of centralized allocation of indivisible objects in multiple markets. Each market may be interpreted either as a different type of object or as a different period. We show that every allocation rule that is strategy-proof, Pareto-efficient and nonbossy is a sequential dictatorship. The result holds for an arbitrary number of agents and for any preference domain that contains the class of lexicographical preferences.

1. Introduction

A central planner often faces the task of distributing indivisible objects to the agents. For example, municipalities assign public houses to families, education departments allocate students to public schools, and firms allocate projects among workers. The problem of assigning indivisible objects to agents when monetary transfers are not allowed has been widely studied from many different perspectives. Pápai (2000), in particular, shows that the only way to implement a Pareto-efficient allocation with a rule that is strategy-proof, nonbossy and reallocation-proof is through the use of a *hierarchical exchange rule*.[1] Pycia and Ünver (2011) further show that the only rules that are strategy-proof, nonbossy and Pareto-efficient are the trading cycles rules. Not only are these results of theoretical importance, but they also provide important guidance for practitioners and policy makers.[2]

We study the centralized allocation problem that takes place in multiple markets, where each market may be interpreted either as a different type of object or as a different period. Indeed, in reality agents are often involved in more than one assignment problem at one time; people who participate in the allocation of public housing, for example, might also have their children enrolled in public schools. In the US there are more than one thousand federally-funded benefit and assistance programs, many of which involve the assignment of indivisible objects. Moreover, a single family may be eligible for many of these programs at the same time.[3] In Brazil there is a comprehensive social program called *Bolsa Familia*, which is a conditional cash transfer program that benefits over 11 million families. The *Bolsa Familia* unified several separate clearinghouses that were already in place. Precisely, it

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1 A hierarchical exchange rule is a generalization of the top trading cycles allocation rule and can be described as follows. In the first stage, the planner distributes the objects to the agents; in particular, some agents might receive multiple objects while others might receive none. Then, the top trading cycles algorithm is applied, with each agent pointing to her preferred object and each object pointing to its owner. The agents who form a cycle receive the objects they pointed to. The non allocated objects whose owners left in the first stage are inherited by the remaining agents and the top trading cycles algorithm is applied again. The procedure is repeated until all agents are assigned an object.

2 For example, on April 16, 2012, it was announced that the New Orleans Recovery School District would utilize a version of the top trading cycles allocation rule as the allocation rule for the centralized enrollment of children in public schools (Vanacore, 2012).

3 The website *benefits.gov* (formerly GovBenefits.gov) is a partnership of seventeen federal agencies as well as other governmental agencies that provides a centralized source of information for many of these assistance programs.

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unified the following existing social programs: National School Allowance Program, the Food Card Program, the Food Allowance Program, and the Child Labor Eradication Program.[4]

Additionally, our results apply to dynamic matching problems. One example of dynamic matching previously studied in the literature is the allocation of new physicians in the United Kingdom, where each young doctor applies for two successive positions: a medical post and a surgical post (Roth, 1991; Irving, 1998). Another illustrative example is the allocation of courses among the faculty of a department in which each professor teaches one undergraduate and one graduate course. The important market design problem known as the school choice problem might be studied as a dynamic matching problem if (i) student mobility is taken into account, or (ii) sibling priorities are considered. Finally, Kennes et al. (2014) introduced the dynamic matching problem of allocating young children to public day care centers.

In our model, there are n agents and two (or more) markets, and each agent must be assigned at most one object from each market. Agents have preferences over the different bundles, where a bundle is a vector consisting of one object per market. We restrict our attention to the cases in which markets are *independent*, by which we mean that the set of objects available in a particular market is exogenous and not affected by the other markets.

In environments with multiple markets, there might be scope for a mutually beneficial trade between agents even if the allocation is Pareto-efficient within each market. This raises the question of our paper: how to implement a Pareto-efficient outcome in a multiple-market problem?

In our main result (Theorem 2) we show that the set of rules that are strategy-proof and nonbossy and that implement a Pareto-efficient allocation are the sequential dictatorships. These rules generalize the serial dictatorship rule in that the order of the agents who choose the objects might be a function of the choices made previously by the other agents.[5] Despite its wide use in the literature, we feel that it is important to justify the use of the nonbossiness axiom in our formulation. First, a rule that fails this axiom is susceptible to coalitional deviations, implying that it might be problematic for implementing it. In addition, from a normative point of view, a bossy mechanism might be considered as unfair, since an agent might be able to dictate others' allocations without changing her own allocation.

We first introduce a novel class of preferences, which we call *(generalized) lexicographic preferences*. We then prove that our result holds for any preference domain that includes the domain of lexicographic preferences. In particular, it holds for the class of separable preferences, a widely used domain when agents demand more than one object. By using the class of lexicographic preferences we are able to contrast our results more sharply with Pycia and Ünver (2011)'s result on single market allocations. In our preference domain, all objects from all markets are ranked for each individual under a single ranking. This means that the cardinality of the set of preferences in our multi-market environment is slightly smaller than the one in a single market case which pools the objects from all the markets. Thus, the fact that our set of rules is much narrower than in the single market case is not due to the increased cardinality of the domain of preferences, but it follows from the specific feature of multiple markets that each individual demands more than one object. From the technical perspective, we believe that the domain of lexicographical preferences will be useful in other studies in which agents demand more than one objects, due to the tractability of this class of preferences.

This paper is related to the literature on centralized allocation of multiple objects. Pápai (2001), Ehlers and Klaus (2003), and Hatfield (2009) also obtain the same characterization result as ours, but in different settings. Hatfield (2009) studies a model in which each agent must be allocated an exact number of objects, which he refers to as fixed *quotas*, from one pool of objects. Our model is related to Hatfield's work in the sense that two markets in our model might be interpreted as a quota of two goods for every individual. However, in our setting any two objects that an individual can be allocated must be drawn from different pool of objects. In addition, the smallest domain the above-mentioned studies consider is the one of separable preferences. Our domain of lexicographical preferences is smaller than that of separable preferences.

To the best of our knowledge, this is the first paper that provides a complete characterization of centralized allocation in multiple markets without an endowment structure. Konishi et al. (2001) considered the multi-type allocation problem,[6] but in their work each agent is initially endowed with one object—as in the economy proposed by Shapley and Scarf (1974). Konishi et al. (2001) show that the core may be empty in these multi-type Shapley–Scarf economies and also that there are no Pareto-efficient, individually rational, and strategy-proof rules. Here, since we do not assume an initial endowment structure, we do not impose the individual rationality constraint, which plays a crucial role in their results.

Nguyen et al. (2014) work on a similar problem of allocating multiple objects – from different markets – to different agents. They provide a mechanism, which generalizes the Probabilistic Serial mechanism (see Bogomolnaia and Moulin (2001)) by having the mechanism return probability shares over the different bundles. They prove that their mechanism is efficient, envy-free and asymptotically strategy-proof. While their matching model is more general than ours, their axioms are not the same as ours, in particular they focus on asymptotic strategy-proofness, for example.

This paper is organized as follows. In the following section we describe the model and state its main assumptions. In Section 3, we describe and define an allocation rule and its main properties. In Section 4, we describe the sequential dictatorship. We prove our main result (Theorem 2) in Section 5. Finally, we conclude the paper in Section 6. In the Appendix we include the proof of the special case of two agents and two goods in each market as well as the general proof of Theorem 2 for the case in which the number of agents is greater than two.

2. Model

Let $N = \{1, \dots, n\}$, where $n < \infty$, be the set of agents. There are two types of indivisible objects, A and B, which also stand for the sets of the respective type objects. We refer to a pair $(a, b) \in A \times B$ as a bundle. For convenience, we assume that an artificial null object, 0, is in both sets A and B. Throughout the paper, we assume that $|A \setminus \{0\}| \geq n$ and $|B \setminus \{0\}| \geq n$, i.e., there are enough A- and B-objects to distribute to the agents. An allocation $x = (x_1, \dots, x_n)$ is a list of the assignments for the n agents, where $x_i \in A \times B$. If $x_i = (a, b)$, then agent i is assigned the bundle (a, b). We write x_i^A (x_i^B) to denote the A-object (B-object) that agent i obtains under allocation x. We refer to $x^A = (x_i^A)_{i \in N}$ and $x^B = (x_i^B)_{i \in N}$ as the A- and B-allocation, respectively. An allocation x is feasible if no object (except the null object) is assigned to more than one agent.

[4] More details can be obtained directly from the official website: http://www.planalto.gov.br/ccivil_03/_ato2004-2006/2004/lei/l10.836.htm.

[5] In the single-market case, the sequential dictatorship rules are special cases of pápai's (2000) hierarchical exchange rules.

[6] See Klaus (2008) for further reference.