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# Equilibrium theory under ambiguity\*

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ABSTRACT

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### 1. Introduction

Modeling the market with uncertainty is of important academic significance and realistic value in economics as most decision making is made under uncertainty. Towards this direction, the Arrow-Debreu "state contingent model" allows the state of nature of the world to be involved in the initial endowments and payoff functions, which is an enhancement of the deterministic general equilibrium model of Arrow-Debreu-McKenzie. According to Arrow-Debreu, agents make contacts ex ante (in period one) before the state of nature is realized and once the state is realized (in period two) the contract is executed and consumption takes place. The issue of incentive compatibility does not arise in this model, as all the information is symmetric. However, for the state contingent model to make sense one must assume that there is an exogenous court or government that enforces the contract ex post, otherwise agents may find it beneficial to renege. Radner (1968, 1982) extended the analysis of Arrow and Debreu by introducing asymmetric (differential) information. In particular, each agent is now characterized by his own private information, a random initial endowment, a random utility function and a prior. The private information is modeled as a partition of a finite state space and the allocation of each agent is assumed to be measurable with respect to his own private information. This means that each agent

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We extend the classical results on the Walras-core existence and equivalence to an ambiguous asymmetric information economy; that is, an economy where agents maximize Maximin Expected Utility (MEU). The interest of considering ambiguity arises from the fact that, in the presence of MEU decision making, there is no conflict between efficiency and incentive compatibility (contrary to the Bayesian decision making). Our new modeling of an ambiguous asymmetric information economy necessitates new equilibrium notions, which are always efficient and incentive compatible.

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only knows the atom of his partition including the true state, but cannot distinguish those states within the same atom when making decisions. The Walrasian equilibrium notion in this model is called 'Walrasian expectations equilibrium', or WEE in short. Along this line, Yannelis (1991) proposed a core concept, which is called private core.<sup>1</sup>

The Walrasian expectations equilibrium and private core share some interesting properties (in fact, the Walrasian expectations equilibrium is a strict subset of the private core): without the assumption of free disposal, whenever agents are Bayesian expected utility maximizers and allocations are private information measurable, the two above notions are both Bayesian incentive compatible and private information measurable efficient (see Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994)). However, these solution concepts are only efficient in the second best sense; that is, they are only private information measurable efficient allocations and may result in a possible welfare loss (recall that from Holmstrom and Myerson (1983), we know that with the Bayesian expected utility it is not possible to have allocations which are both first best efficient and also incentive compatible). The existence of WEE in a free disposal economy can be found in Radner (1968, 1982). However, the free disposal WEE allocations may be not incentive compatible (see Glycopantis and Yannelis (2005)). Furthermore, if we require non-free disposal, then a WEE may not exist (see Einy and Shitovitz (2001)). Therefore, a natural question arises:





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<sup>&</sup>lt;sup>1</sup> For a recent treatment of general equilibrium with asymmetric information, see the books Glycopantis and Yannelis (2005) and Marakulin (2013a).

Can one find an appropriate framework in the asymmetric information economy such that the existence of equilibrium and core notions continues to hold and furthermore, these notions are both incentive compatible and first best efficient?

A crucial assumption in the frameworks of Radner (1968. 1982) and Yannelis (1991) is that agents maximize Bayesian expected utilities. Nevertheless, from Ellsberg (1961) (see also de Castro and Yannelis (2013)), there is a huge literature which criticizes the Bayesian paradigm and explores the non-expected utility theory. The maximin expected utility of Gilboa and Schmeidler (1989) is one of the successful alternatives. Indeed, recently de Castro et al. (2011, 2014) and de Castro and Yannelis (2013) applied the maximin expected utility to an asymmetric information economy with a finite number of states of nature,<sup>2</sup> and introduced various core and Walrasian equilibrium notions. With the maximin expected utilities, agents take into account the worst possible state that can occur and choose the best possible allocations. de Castro et al. (2011) proved that the ex ante equilibrium and core notions based on the maximin expected utility, which are called maximin expectations equilibrium (MEE) and maximin core (MC) therein, are incentive compatible in the economy without free disposal. Moreover, it is noteworthy that since the allocations are not required to be measurable with respect to agents' private information, MEE and MC allocations are also first best efficient. Therefore, the conflict between efficiency and incentive compatibility is solved in this new approach. More importantly, de Castro and Yannelis (2013) showed that the conflict of incentive compatibility and first best efficiency is inherent in the standard expected utility decision making (Bayesian) and it is resolved only when agents maximize the maximin expected utility (MEU). In particular, they proved that the MEU is a necessary condition for efficient allocations to be incentive compatible. The above work implies the fact that one has to work with MEU if the first best efficiency is desirable. As a result, a natural question arises:

Can one obtain the classical core–Walras existence and equivalence results for asymmetric information economies where agents are ambiguous (*i.e.*, MEU maximizers) and also the state space is not necessarily finite?

An affirmative answer to this question is of great importance because not only this way one develops a new equilibrium theory where there is no conflict between efficiency and incentive compatibility, but also such positive results could become the main tool for applications in other fields of economics.

The first aim of this paper is to prove the existence of the maximin expectations equilibrium and maximin core in a nonfree disposal economy with countably many states of nature.<sup>3</sup> Since there is a countable number of states in the economy, the allocations are infinite dimensional. An advantage of the ambiguous economy modeling is that it allows us to view an asymmetric information economy as a deterministic economy with infinite dimensional commodity spaces. Thus, we can directly apply known results in the literature to obtain the existence of maximin expectations equilibrium.<sup>4</sup> As a corollary, we obtain that the consistency between incentive compatibility and efficiency also holds with a countable number of states.

The second aim of the current paper is to prove a core equivalence theorem for an economy with asymmetric information where agents are ambiguous (*i.e.*, maximize MEU). In a finite agent framework and complete information. Debrey and Scarf (1963) considered a sequence of replicated economy and showed that the set of non-blocked allocations in every replicated economy converges to the set of Walrasian equilibria. In Section 4, we follow the Debreu-Scarf approach and establish a similar equivalence result for an equal treatment economy with asymmetric information, a countable number of states and MEU preferences. In an atomless economy with complete information, Schmeidler (1972), Grodal (1972) and Vind (1972) improved the core-Walras equivalence theorem of Aumann (1964), by showing that if an allocation is not in the core, then it can be blocked by a non-negligible coalition with any given measure less than 1. Hervés-Beloso et al. (2005a,b) first extended this result to an asymmetric information economy with the equal treatment property and with an infinite dimensional commodity space by appealing to the finite dimensional Lyapunov's theorem. Bhowmik and Cao (2012, 2013a) obtained further extensions based on an infinite dimensional version of Lyapunov's theorem. All the above results rely on the Bayesian expected utility formulation and therefore the conflict of efficiency and incentive compatibility still holds despite the non atomic measure space of agents.<sup>5</sup> Our Theorem 6 is an extension of Vind's theorem to the asymmetric information economy with the equal treatment property and a countable number of states of nature, where agents behave as maximin expected utility maximizers. Thus, our new core equivalence theorem for the MEU framework, resolves the inconsistency of efficiency and incentive compatibilitv

Finally, we provide two characterizations for maximin expectations equilibrium. In the complete information economy with finite agents, Aubin (1979) introduced a new approach that at a first glance seems to be different from the Debreu-Scarf; however one can show that they are essentially equivalent. Aubin considered a veto mechanism in the economy when a coalition is formed; in particular, agents are allowed to participate with any proportion of their endowments. The core notions defined by the veto mechanism, is called Aubin core and it coincides with the Walrasian equilibrium allocations. The approach of Aubin has been extended to an asymmetric information economy to characterize the Walrasian expectations equilibrium (see for example Graziano and Meo (2005), Hervés-Beloso et al. (2005b) and Bhowmik and Cao (2013a)). Another approach to characterize the Walrasian expectations equilibrium is due to Hervés-Beloso et al. (2005a,b). They showed that the Walrasian expectations equilibrium allocation cannot be privately blocked by the grand coalition in any economy with the initial endowment redistributed along the direction of the allocation itself. This approach has been extended to a pure exchange economy with an atomless measure space of agents and finitely many commodities, and an asymmetric information economy with an infinite dimensional commodity space (e.g., see Hervés-Beloso and Moreno-García (2008), Bhowmik and Cao (2013a,b)). Our Theorems 2 and 3 extend these two characterizations to the asymmetric information economy with ambiguous agents and with countably many states of nature.

The paper is organized as follows. Section 2 states the model of ambiguous asymmetric information economies with a countable number of states and discusses main assumptions.

<sup>&</sup>lt;sup>2</sup> MEU is first applied to a general equilibrium model of an asymmetric information economy by Correia-da-Silva and Hervés-Beloso (2009). They proved the existence of the ex ante Walrasian equilibrium in an asymmetric information economy with maximin preferences and a finite state space. However, their setup is different from ours and they do not consider the issue of incentive compatibility; see also Correia-da-Silva and Hervés-Beloso (2012, 2014).

<sup>&</sup>lt;sup>3</sup> For a general equilibrium model with countably many states, see, for example, Hervés-Beloso et al. (2009).

<sup>&</sup>lt;sup>4</sup> On the contrary, one cannot readily convert an asymmetric information economy with Bayesian expected utility maximizers to an economy with infinite dimensional commodity spaces due to the restriction of the private information measurability requirement. For some papers with infinite dimensional commodity spaces, see, for example, Bewley (1972) and Podczeck and Yannelis (2008).

 $<sup>^5</sup>$  As the work of Sun and Yannelis (2008) indicates, even with an atomless measure space of agents we cannot guarantee that WEE allocations are incentive compatible.

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