Journal of Mathematical Economics 61 (2015) 96–103

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Random scheduling with deadlines under dichotomous preferences

İbrahim Barış Esmerok

Rice University, Department of Economics, MS#22, 6100 S. Main Street, Houston, TX 77005-1892, United States

ARTICLE INFO

Article history:
Received 9 November 2014
Received in revised form
28 April 2015
Accepted 6 August 2015
Available online 20 August 2015

Keywords:
Random assignment
Scheduling with deadlines
Probabilistic consistency
Efficient uniform rule

ABSTRACT

A group of individuals share a deterministic server which is capable of serving one job per unit of time. Every individual has a job and a cut off time slot (deadline) where service beyond this slot is as worthless as not getting any service at all. Individuals are indifferent between slots which are not beyond their deadlines (compatible slots). A schedule (possibly random) assigns the set of slots to individuals by respecting their deadlines. We only consider the class of problems where for every set of relevant slots (compatible with at least one individual) there are at least as many individuals who have a compatible slot in that set: we ignore the case of underdemand. For this class, we characterize the random scheduling rule which attaches uniform probability to every efficient deterministic schedule (efficient uniform rule) by Pareto efficiency, equal treatment of equals, and probabilistic consistency (Chambers, 2004). We also show that a weaker version of the probabilistic consistency axiom is enough to achieve our result. Finally we show that efficient uniform rule is strategyproof.

1. Introduction

A group of individuals share a deterministic server which is capable of serving one job per unit of time. Every individual has a job and a cut off time slot (deadline) where service beyond this slot is as worthless as not getting any service at all. The compatible slots for an individual are the ones which are not beyond his deadline and he is indifferent between any two of them. Therefore, the preferences are dichotomous. A schedule (possibly random) assigns the set of slots to individuals by respecting their deadlines. We only consider the class of problems where for every set of relevant slots (compatible with at least one individual) there are at least as many individuals who have a compatible slot in that set. We are ignoring the case of underdemand which is not interesting. Under an efficient (random) schedule all (relevant) slots are assigned to individuals with probability one in this class.

As an example of our problem consider the case where individuals apply for a visa. Each individual has a personal deadline to receive it. Individuals get fully satisfied if they receive their visa before their deadlines, receiving a late visa has no value. The problem is interesting when the total number of available visas falls short of the total demand. In such cases randomly allocating visas is a common method, for example H1B work visas in the USA is allocated by a lottery if they are overdemanded. As an other example consider the case where a group graduate students share a common resource (a lab, a supercomputer) in a university. Such resources are used by taking turns, all students only care about using the resource before it is too late for them.

The central axiom for our analysis is consistency. In a deterministic environment, consistency is defined as follows: apply a (scheduling) rule to a (scheduling) problem. Then pick an individual and the slot which is assigned to him (if there is any). Reapply the same rule to the subproblem which retains the remaining relevant slots (all relevant slots in the initial problem may still be available) and individuals with induced preferences. Consistency recommends that the allocation for the subproblem should coincide with the allocation for the original problem in its relevant components. If this is true for all problems and all individuals then the rule is consistent. Chambers (2004) introduces a probabilistic version of consistency axiom for a probabilistic assignment model. In the probabilistic analogue of consistency we again apply a (probabilistic) rule to a problem. Pick an individual and a slot such that our rule has attached a positive probability for him to get that slot. Re-apply the same rule to the subproblem which retains the remaining slots and individuals with induced preferences. Probabilistic consistency recommends that the solution of the subproblem should be the Bayesian update of the original problem's solution given that the individual receives that particular slot. If it is true for all problems, all individuals, and all slots that the individual gets with positive probability (considering also the no match case) then the rule meets probabilistic consistency. Note that we are only interested in efficient rules therefore, we are not considering subproblems where all individuals from the original problem are present but a slot is removed.

E-mail address: esmerok@rice.edu.

http://dx.doi.org/10.1016/j.jmateco.2015.08.001

As our main result we show that the random scheduling rule which attaches equal probability to every efficient deterministic schedule (efficient uniform rule) is the only rule which meets efficiency, equal treatment of equals, and probabilistic consistency. We also provide an alternative definition of the efficient uniform rule by a simple ball drawing algorithm. Let slot d_N be the highest indexed relevant slot. Put a ball to an urn for every individual who demands slot d_N, draw a ball with uniform probability and assign slot d_N to the associated individual. Keep the rest of the balls in the urn and add a ball for every individual who demands slot $d_N - 1$ and does not demand slot d_N to the urn. Draw a ball with uniform probability to assign slot $d_N - 1$. Repeat this process to assign all relevant slots. This algorithm is well defined: there is always a ball to be drawn in the urn, and it coincides with the efficient uniform rule.

The rest of the paper is organized as follows. In Section 2 we discuss the relation of our work to the existing literature. In Section 3 we introduce our formal model and main axioms. In Section 4 we define the efficient uniform rule and present our main characterization result. In Section 5 we discuss a weaker version of probabilistic consistency, and we show that the weaker axiom is essentially enough to obtain our result. We also discuss how our results relate with the (random version of) classical consistency axiom in a more general setting. In Section 6 we show that the efficient uniform rule is strategyproof. Section 7 concludes.

2. Relation to literature

Cres and Moulin (2001) and Bogomolnaia and Moulin (2002) also consider (random) scheduling problems with deadlines, different from our work they assume that individuals prefer early service. Their model is a special case of (random) assignment problems under strict preferences (see Bogomolnaia and Moulin, 2001). Under the assignment with strict preferences model, Chambers (2004) introduces the probabilistic consistency axiom, and he shows that the combination of probabilistic consistency and equal treatment of equals is not compatible with (any kind of) efficiency. Our model is a special case of the model in Bogomolnaia and Moulin's (2004) random matching under dichotomous preferences paper. The following matrix presents the relations between these papers. We will next describe the existing work in more detail.

	Strict preferences	Dichotomous preferences
Random assignment/ matching	Bogomolnaia and Moulin (2001)	Bogomolnaia and Moulin (2004)
	Chambers (2004)	
Random scheduling with deadlines	Cres and Moulin (2001)	Our paper
	Bogomolnaia and Moulin (2002)	

For the scheduling with deadlines under strict preferences model, Cres and Moulin (2001) introduce the probabilistic serial solution and they show that the probabilistic serial solution improves upon the random priority solution. Bogomolnaia and Moulin (2002) provide axiomatic characterizations of the probabilistic serial solution for this model.

Bogomolnaia and Moulin (2001) define the probabilistic serial solution for the more general model of assignment with strict preferences. Chambers (2004) shows that the combination of probabilistic consistency and equal treatment of equals characterizes the random rule which attaches equal probability to all deterministic assignments (efficient or not) for the assignment under strict preferences model when the number of individuals is equal to the number of objects and individuals are compatible with all objects. Therefore, adding any kind of efficiency axiom to this group of axioms leads to an impossibility result.

In the random matching under dichotomous preferences model of Bogomolnaia and Moulin (2004), there are two (gender) groups: men and women. Each person see people from the other gender group as acceptable or not. By taking one side of the market passive (time slots) and making an assumption over the (dichotomous) preferences of the individuals (in the active side) we reach to our model. Bogomolnaia and Moulin (2004) defines the egalitarian solution for their model. The egalitarian solution targets to equalize the probability of having an acceptable partner for all individuals. They show that the expected utility profile of the egalitarian solution is Lorenz optimal.

3. Model and main axioms

Let $\mathcal{N}$ be a potential set of individuals and let $N \subseteq \mathcal{N}$ be a finite set of individuals with cardinality n. We assume that there are infinitely many individuals in $\mathcal{N}$. Each individual $i \in N$ has a job with identical length. Individuals in N share a deterministic server which is capable of serving one job per unit of time. Every $i \in N$ has a deadline, $d_i \in \mathbf{N}_+ = \{1, 2, 3, \dots\}$ meaning that for individual i getting service beyond d_i is as worthless as not getting service at all. We call $(N, d \in \mathbf{N}_+^n)$ a scheduling problem with deadlines and we refer to it simply as d whenever it is convenient.

Individual i strictly prefers to be scheduled to a slot in $\{1, \dots, d_i\}$ to not being scheduled at all and he is indifferent between any two slots in $\{1, \dots, d_i\}$. Hence, we speak of dichotomous preferences where $\{1, \dots, d_i\}$ is the set of slots compatible with individual i.

Denote by $\mathcal{D}$ the set of scheduling problems. For any $d \in \mathcal{D}$, $\bigcup_{i \in N}\{1, \dots, d_i\}$ is the set of relevant slots. Denote the highest indexed relevant slot by $d_N = \max_{i \in N} d_i$. A deterministic schedule for d is a mapping $s : \{1, \dots, d_N\} \to N \cup \{\emptyset\}$ such that for any distinct $a, b \in \{1, \dots, d_N\}$, if $s(a) = s(b)$ then $s(a) = s(b) = \emptyset$ and for any $a \in \{1, \dots, d_N\}$ such that $s(a) \neq \emptyset$, $a \leq d_{s(a)}$. Assigning slot a to $\emptyset$ means that slot a is left vacant; moreover we only assign slots to individuals who value them. We denote the set of deterministic schedules for d by $\mathcal{S}(d)$. In conjunction with deterministic schedule s we define $s^{-1} : N \to \{1, \dots, d_N\} \cup \{\emptyset\}$ as follows: for all $i \in N$ and for all $a \in \{1, \dots, d_i\}$, if $s(a) = i$ then $s^{-1}(i) = a$; fix $i \in N$, if for all $a \in \{1, \dots, d_i\}$, $s(a) \neq i$ then $s^{-1}(i) = \emptyset$.

A random schedule π is a lottery on $\mathcal{S}(d)$, and $\Delta\mathcal{S}(d)$ is the set of all random schedules. For $s \in \mathcal{S}(d)$ let $p^\pi(s)$ be the probability attached to s by $\pi \in \Delta\mathcal{S}(d)$. Given $\pi \in \Delta\mathcal{S}(d)$, for individual i the canonical utility representation is $u_i(\pi) = \sum_{s \in \mathcal{S}(d)} p^\pi(s) u_i(s)$ where $u_i(s) = 1$ if there exists a slot $a \in \{1, \dots, d_i\}$ such that $s(a) = i$ and $u_i(s) = 0$ otherwise.

Definition 1. A rule r is a mapping $d \to \Delta\mathcal{S}(d)$ and $[r(d)]$ is the support of r for problem d.

Efficiency:

We start by exploring efficient deterministic schedules. Our efficiency axiom is the familiar Pareto optimality. We denote the set of efficient deterministic schedules for d by $\mathcal{E}\mathcal{S}(d)$ and the set of individuals served by schedule $s \in \mathcal{S}(d)$ by

$$I^d(s) = \{i \in N \mid \exists a \in \{1, \dots, d_i\} : s(a) = i\}.$$

Since preferences are dichotomous, $I^d(s) \subset I^d(s')$ if and only if s' Pareto improves upon s therefore, $s \in \mathcal{E}\mathcal{S}(d)$ if and only if $I^d(s)$ is inclusion maximal:

$$s \in \mathcal{E}\mathcal{S}(d) \Leftrightarrow \forall s' \in \mathcal{S}(d), I^d(s) \not\subset I^d(s').$$