Journal of Mathematical Economics 61 (2015) 119–129

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

"Agreeing to disagree" type results under ambiguity

Adam Dominiak [a], Jean-Philippe Lefort [b,*]

[a] Department of Economics, Virginia Polytechnic Institute & State University (Virginia Tech), Blacksburg, VA 24061-0316, USA

[b] Université Paris-Dauphine, LEDa Place du Maréchal de Lattre de Tassigny, 75 775 Paris cedex 16, France

ARTICLE INFO

Article history:
Received 22 December 2014
Received in revised form
27 July 2015
Accepted 12 August 2015
Available online 3 September 2015

Keywords:
Ambiguity
Choquet expected utility
Unambiguous events
Updating
Asymmetric information
Agreement theorems

ABSTRACT

This paper characterizes conditions under which it is impossible for non-Bayesian agents to "agree to disagree" on their individual decisions. The agents are Choquet expected utility maximizers. Whenever each agent's information partition is composed of unambiguous events in the sense of Nehring (1999), then the agents cannot disagree on the common knowledge decisions, whether these decisions are conditional capacities or conditional Choquet expectations. Conversely, an agreement on conditional Choquet expectations, but not on conditional capacities, implies that each agent's private information must consist of Nehring-unambiguous events. These results indicate that under ambiguity – contrary to the standard Bayesian framework – asymmetric information matters, and it can explain differences in common knowledge decisions due to the ambiguous nature of the agents' private information.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

In his celebrated article "Agreeing to Disagree", Aumann (1976) challenged the role that asymmetric information plays in the context of interpersonal decision problems under uncertainty. Presupposing that agents are Bayesian and share an identical prior probability distribution, Aumann proved that the agents cannot "agree to disagree" on the posterior probabilities they attach to some event. That is, when the agents' posteriors are common knowledge then these posteriors must coincide even though they may be conditioned on diverse information. Consequently, the common knowledge posteriors do not reveal the differences in information about events which the Bayesian agents may have observed. This paper scrutinizes the role of asymmetric information among non-Bayesian agents. It is shown that differences in common knowledge posterior decisions are possible due to the ambiguous character of private information.

Under the Bayesian paradigm, Aumann's impossibility result has been extended to more abstract decisions such as posterior expectations, or optimal contingent plans (see Geanakoplos and Sebenius, 1983; Milgrom and Stokey, 1982; Bacharach, 1985; Rubinstein and Wolinsky, 1990). These results, sometimes referred to as probabilistic agreement theorems, "suggested that asymmetric information, then a new but rapidly growing topic of microeconomic research, had less explanatory power that might be thought" (Morris, 1995, p. 229). Differences in individual decisions cannot be explained *solely* on the basis of differences in agents' private information. This constraint has also been seen as a problem for the theory of speculative markets: informational differences *alone* cannot explain the existence of the widely observed speculative behavior. In order to overcome these limitations, two main approaches have been suggested in the literature. In the first approach, Morris (1994, 1995) advocates discarding the "commonness" assumption of prior probabilities; in the second approach, suggested by Monderer and Samet (1989), the notion of "common knowledge" is replaced by a weaker concept of "common p-beliefs". Both approaches continue to maintain the Bayesian doctrine. However, an alternative approach is suggested in this paper. We shall maintain the assumption of common prior beliefs as well as the notion of common knowledge. Instead, the "additivity" property of subjective beliefs is abandoned by allowing the agents to be non-Bayesian in spirit of the *Choquet expected utility theory* axiomatized by Schmeidler (1989). In Schmeidler's theory, subjective beliefs are represented by a normalized and monotone (but-not-necessarily-additive) set function called *capacity*. The notion of capacity allows one to incorporate *ambiguity* into the decision-making process.

The main goal is to examine how ambiguity will affect the outcome of common knowledge posterior decisions. There is a finite group of agents. The agents face a dynamic decision

* Corresponding author.
E-mail addresses: dominiak@vt.edu (A. Dominiak), jean-philippe.lefort@dauphine.fr (J.-P. Lefort).

http://dx.doi.org/10.1016/j.jmateco.2015.08.005
0304-4068/© 2015 Elsevier B.V. All rights reserved.

problem under ambiguity. All agents share a common prior capacity on an algebra of events generated by a finite set of states. Each agent is endowed with a partition over the set of states that represents his private information. There are two stages of planning: ex-ante and interim. In the ex-ante stage, all agents have identical information. In the interim stage, each agent receives a private signal and incorporates it by revising his unconditional preferences. Conditional preferences are derived by updating the unconditional capacity and keeping the utility function unchanged. There are many reasonable rules for updating non-additive beliefs, with Bayes' rule being only one example (see Gilboa and Schmeidler, 1993). However, our results do not depend upon which updating rule is utilized. It is only assumed that updating rules respect *consequentialism*, a property of preferences introduced by Hammond (1988, 1989). Consequentialism requires that conditional preferences are only affected by the conditioning events (i.e., private information in our setup). Counter-factual events, as well as the past decision history, are immaterial for posterior choices (see Hanany and Klibanoff, 2007, 2009). Once conditional preferences are generated, the agents announce their individual decisions. An agreement designates a situation where it is impossible that the agents' posterior decisions are common knowledge and not the same. The posterior decisions focused on in this paper are conditional capacities for some fixed event and conditional Choquet expectations for a given action.

The first objective is to find the properties of the events representing each agent's private information that preclude opportunities for disagreements on posterior decisions. The analysis starts with the assumption that agents' partitions are compound of events that are *unambiguous*, while other events may be ambiguous. In the Bayesian framework, where the probabilistic agreement theorems are formulated, all uncertain events are revealed to be unambiguous. However, in non-Bayesian setups some events may be perceived as unambiguous and other ones as ambiguous. Several notions of subjectively revealed unambiguous events have been proposed and studied in the literature, e.g., by Nehring (1999); Epstein and Zhang (2001); Zhang (2002); Ghirardato et al. (2004); and Kopylov (2007). It is shown that Nehring's notion of unambiguous events is sufficient for the impossibility of "agreeing to disagree". That is to say, when each agent's private information is represented by Nehring-unambiguous events then common knowledge posterior decisions must always coincide. The posterior decisions can be based on conditional capacities or conditional Choquet expectations. Note that this result is an adaptation of the existing literature establishing the link between dynamic properties of Choquet preferences and the notion of Nehring-unambiguous events in the context of "agreeing to disagree" problems (see Remark 1 and Dominiak and Lefort, 2011). However, as soon as one departs from the Nehring's notion then disagreements on common knowledge posterior decisions may occur. Under a weaker notion of unambiguous events suggested by Zhang (2002), the agents may "agree to disagree" on the values of their conditional capacities for some event. We exemplify a disagreement between two agents whose private information is perceived as Zhang-unambiguous events.

The second objective is to establish the converse result. Situations are considered where disagreements are impossible to occur. An immediate question that arises in this context is the following: can one infer something about the properties of events representing agents' private information given that the agents cannot "agree to disagree" on posterior decisions? In principle, the question can be answered in the affirmative. However, what may be inferred about the nature of the agents' private information depends on the type of decisions on which the agents "agree to agree". When agents cannot "agree to disagree" on their values of conditional capacities, we shall argue that nothing can be inferred about the properties of the conditioning events. One can always find a capacity distribution over an algebra of events and a suitable updating rule for prior beliefs so that an agreement on conditional capacities will hold true. Notwithstanding, the events in each agent's information partition will neither be perceived as Nehring-unambiguous nor as Zhang-unambiguous events. This argument will be further elaborated upon with the help of a few examples. However, when the agents reach an agreement on conditional Choquet expectations then each agent's private information must consist of Nehring-unambiguous events.

The remainder of the paper is organized as follows. In Section 2, the capacity model is introduced and two notions of unambiguous events are presented. In Section 3, the capacity model is extended to dynamic choice situations. Section 4 recalls the standard epistemic framework used for modeling interpersonal decision problems with differential information. In Section 5, the sufficient condition for the impossibility of "agreeing to disagree" on posterior decisions is established. This section concludes by exemplifying circumstances under which agents may "agree to disagree" on their conditional capacities. In Section 6, the necessary condition for the impossibility of disagreements on conditional Choquet expectations is derived. Finally, a brief discussion on the meaningfulness of consequentialism in the context of interpersonal decision problems with differential information is provided. The conclusions of this paper are summarized in Section 7.

2. Static Choquet preferences

This section recalls the theory of Choquet expected utility maximization pioneered by Schmeidler (1989) together with two preference-based notions of unambiguously perceived events.

There is a finite set Ω of states. An event E is a subset of Ω. For any $E \subset \Omega$, the complement of E is denoted by E^c. The set of all subsets of Ω is denoted by $\mathcal{A} = 2^{\Omega}$. Subjective beliefs are represented by *capacities*. A capacity $\nu : \mathcal{A} \to \mathbb{R}$ is a normalized and monotone set function: (i) $\nu(\emptyset) = 0$, $\nu(\Omega) = 1$ and (ii) $\nu(E) \leq \nu(F)$ whenever $E \subset F$. Capacities are not required to be additive, although they must satisfy the monotonicity property; "larger" events, with respect to the set inclusion, are regarded as "more likely".

Let X be a set of consequences. A mapping $f : \Omega \to X$ assigning consequences to states is called an action. Let $\mathcal{F}$ be a set of all actions. A subset $\mathcal{B} \subset \mathcal{F}$ is referred to as a set of feasible actions. For a pair of actions $f, g \in \mathcal{F}$ and an event $E \in \mathcal{A}$, $f_E g$ denotes an action that assigns a consequence $f(\omega) \in X$ to ω in E and $g(\omega) \in X$ to ω in E^c. A preference relation $\succcurlyeq$ on the set $\mathcal{F}$ is said to admit *Choquet expected utility* representation if there exist a utility function $u : X \to \mathbb{R}$ and a capacity ν on $\mathcal{A}$ such that for any $f, g \in \mathcal{F}$:

$$f \succcurlyeq g \Leftrightarrow \int_{\Omega} u \circ f \, d\nu \geq \int_{\Omega} u \circ g \, d\nu. \tag{1}$$

The family of Choquet preferences was originally axiomatized by Schmeidler (1989) in the setup of Anscombe and Aumann's (1963). In the Savage-style framework, Choquet preferences have been behaviorally underpinned by Gilboa (1987), Wakker (1989b), Sarin and Wakker (1992), Nakamura (1990), and Chew and Karni (1994).

The expectations in (1) take the form of *Choquet integrals*. For an $f \in \mathcal{F}$, let $E_1, \ldots, E_n$ be the partition ordered from the most to the least favorable events (i.e., $u\big(f(E_1)\big) \geq \cdots \geq u\big(f(E_n)\big)$). The Choquet integral of f with respect to ν and u is defined to be

$$\int_{\Omega} u \circ f \, d\nu = \sum_{j=1}^{n} u\big(f(E_j)\big) \times \Big[\nu\big(E_1, \ldots, E_j\big) - \nu\big(E_1, \ldots, E_{j-1}\big)\Big], \tag{2}$$

Download English Version:

https://daneshyari.com/en/article/965172

Download Persian Version:

https://daneshyari.com/article/965172

Daneshyari.com