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Utilizing belief functions for the estimation of future climate change

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Abstract

We apply belief functions to an analysis of future climate change. It is shown that the lower envelope of a set of probabilities bounded by cumulative probability distributions is a belief function. The large uncertainty about natural and socio-economic factors influencing estimates of future climate change is quantified in terms of bounds on cumulative probability. This information is used to construct a belief function for a simple climate change model, which then is projected onto an estimate of global mean warming in the 21st century. Results show that warming estimates on this basis can generate very imprecise uncertainty models.

Keywords: Climate change; Climate sensitivity; Imprecise probability; Belief function; Random set; Probability box; Distribution band

1. Introduction

It is widely accepted that a discernible influence of industrial and agricultural emissions of greenhouse gases (GHGs) on the earth's climate exists. Due to human activity, greenhouse gas concentrations in the atmosphere have risen by, to name just a few gases, 35% for carbon dioxide, 250% for methane and 15% for nitrous oxide.

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Empirical evidence for a growing climate change signal is mounting, and all state-of-the-art climate models need the increased absorption of infrared radiation from the earth surface due to growing GHG concentrations to reproduce this signal. Still, uncertainty abounds. How sensitive is the climate to growing GHG concentrations? What amount of greenhouse gases will humankind put into the atmosphere in the 21st century?

We believe that the application of imprecise probability concepts carries the potential to greatly improve the situation in climate change forecasting and integrated assessment of climate change policies. However, an obstacle might be the dynamical nature of climate change models, and the large number of uncertain variables which mostly range over continuous possibility spaces. In this paper, we present an application of belief functions to the estimation of global mean temperature (GMT) change in the 21st century. Belief functions have been widely used with varying interpretations. Luo and Caselton [1], for example, have presented an early application of belief functions to climate change by drawing on Dempster's theory [2] which builds on a basic probability assignment to sets.

We interpret a belief function as lower envelope of a set of probability measures, and try to respect this interpretation throughout the reasoning process. The uncertainty on the climate model parameters is initially quantified by lower and upper cumulative probability distribution functions (CDFs) on the real line. In Section 2, we discuss how this information can be converted to a belief function, combined for different model parameters, and projected onto the model output. In Section 3, we present the simple temperature change model, and construct a joint belief function for its uncertain parameters. In Section 4, the uncertainty in the input values is projected onto an estimate of global mean temperature change. Section 5 concludes the paper. To enhance readability, proofs of formal statements are delegated to Appendix A.

2. Methods

2.1. Basic concepts

We briefly introduce the basic concepts that are used throughout the paper. Consider an uncertain quantity X that enters a model of some causal relationship, e.g., of the link between GHG emissions and GMT. The uncertainty about X shall be described by a lower bounding function $\underline{F}_X : \mathbb{R} \to [0,1]$ and an upper bounding function $\overline{F}_X : \mathbb{R} \to [0,1]$ for a set of CDFs $F_X(x) := P(X \leqslant x)$ on the real line $\mathbb{R}$. The resulting set of probabilities

$$\Gamma_X(\underline{F}, \overline{F}) := \{P | \forall x \in \mathbb{R} \;\; \underline{F}(x) \leqslant P(-\infty, x] \leqslant \overline{F}(x)\} \tag{1}$$

has been called a *distribution band* in the literature [3]. Γ_X is convex, since for any two probabilities $P, Q \in \Gamma_X$ and $\lambda \in (0,1)$, also $\lambda P + (1-\lambda)Q \in \Gamma_X$.

If the lower and upper bounding functions are continuous on the real line, we call Γ_X a *continuous distribution band*. Another important special case is constituted by