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Intertemporal choice with different short-term and long-term discount factors

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ABSTRACT

This paper proposes a new axiomatic model of intertemporal choice that allows for dynamic inconsistency. We weaken the classical assumption of stationarity into two related axioms: stationarity in the short-term and stationarity in the long-term. We obtain a model with two independent discount factors, which is flexible enough to capture different time preferences, including a greater impatience for more immediate outcomes (when a long-term discount factor exceeds a compounded short-term discount factor). Our proposed model can accommodate some experimental results that cannot be rationalized by other existing models of dynamic inconsistency (such as quasi-hyperbolic discounting and generalized hyperbolic discounting).

Discounted utility (Samuelson, 1937) is the most popular model of intertemporal choice. The main behavioral assumption of constant (exponential) discounting is stationarity (Koopmans, 1960, postulate 4, p. 294). Yet, empirical evidence suggests that decision makers may violate stationarity (*e.g.*, Loewenstein and Prelec, 1992, Section II, pp. 574–578). Several generalizations of discounted utility were proposed in the literature including quasi-hyperbolic discounting (Phelps and Pollak, 1968), generalized hyperbolic discounting (Loewenstein and Prelec, 1992), the similarity theory (Rubinstein, 2003), subadditive discounting (Scholten and Read, 2010) and liminal discounting (Pan et al., 2013).

This paper proposes a new model of intertemporal choice that allows for dynamic inconsistency. Our approach is to weaken the classical assumption of stationarity into two related behavioral assumptions: stationarity in the short-term and stationarity in the long-term. We obtain a model with two different discount factors. We can think of one of them as a long-term discount factor and the other—as a short-term discount factor.

Intuitively, our proposed model works as follows. Suppose that time periods are measured in days and seven days (*i.e.*, one week) constitute one short term period. Within each week, a decision maker discounts daily utilities using one (short-term) discount factor. This generates weekly utilities. A decision maker then discounts these weekly utilities using another (long-term) discount factor.

A model with two different discount factors is useful in several applied problems. For example, consider a customer with a line of credit for a certain time period (*e.g.*, one month). The customer must pay a high interest rate if a payment is deferred for a longer period. In this case, it is rather natural to assume that the customer distinguishes between the short term (within one credit period) and the long term. The customer may discount little (or not at all) the time periods falling within one credit period. At the same time, the customer may use a lower discount factor in the long term (for time periods extending beyond the length of the credit period).

As another example, consider a taxpayer whose income/revenue is accounted for within a certain fiscal period (*e.g.*, one year). The taxpayer may differentiate between time periods falling within one fiscal period (the short term) and falling into different fiscal periods (the long term). Under progressive taxation, the taxpayer may use a lower discount factor in the short term since an increased income/revenue within the current fiscal period increases the tax burden. The taxpayer may use a higher discount factor in the long term since income/revenue received in different time periods is taxed with a lower tax rate.

In a related example, consider an organization operating on an annual budget. Again, such a decision maker may use one discount factor in the short term (within one year) when its approved budget funds are known with certainty. The same organization, however, may use a different discount factor in the

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long term (several years) since the availability of budget funds is uncertain/unknown across different budget periods.

Finally, a model with two discount factors is a useful framework in situations where intertemporal impatience is compounded with a certain survival probability. For example, a decision maker may use one discount factor in the short term that reflects his or her intertemporal impatience. The same decision maker may use another discount factor in the long term that reflects his or her perceived survival probability. For individuals, this can be the perception of own mortality rate. For firms, this can be the estimated probability of remaining on the market.

In our proposed model, two discount factors are independent of each other. This creates a flexible framework for capturing different types of time preferences. Specifically, our proposed model can accommodate dynamically consistent preferences (when a long-term discount factor coincides with a compounded short-term discount factor), a greater impatience for immediate outcomes (when a long-term discount factor is greater than a compounded short-term discount factor) and a greater patience, possibly even no discounting at all, within a short term period (when a long-term discount factor is smaller than a compounded short-term discount factor).

The remainder of the paper is organized as follows. Section 1 introduces the mathematical notation and our proposed model. Bleichrodt et al. (2008) recently provided a behavioral characterization (axiomatization) of the discounted utility model of Samuelson (1937). We adopt the framework of Bleichrodt et al. (2008) for characterizing the behavioral properties of our proposed model in Section 2. Section 3 compares our proposed model with other generalizations of discounted utility such as quasi-hyperbolic discounting (Phelps and Pollak, 1968), generalized hyperbolic discounting (Loewenstein and Prelec, 1992), the similarity theory (Rubinstein, 2003) and liminal discounting (Pan et al., 2013). Section 4 applies our proposed model to several behavioral regularities in intertemporal choice (experiment I reported in Rubinstein, 2003 and the common difference effect of Loewenstein and Prelec, 1992). Section 5 concludes.

1. Notation and the model

There is a connected and separable set X. The elements of X are called *outcomes*. An outcome can be a monetary payoff, a consumption bundle, a financial portfolio, a health state etc. A *program* is an infinite sequence of outcomes $\{x_t\}_{t=1}^{\infty}$, where $x_t \in X$ is an outcome received in time period $t \in \mathbb{N}$. The set of all programs is denoted by $\mathbb{P}$.

For a compact notation let $y_T p \in \mathbb{P}$ denote a program that yields outcomes $\{y_1, y_2, \ldots, y_T\} \in X^T$ in the first T periods, for some $T \in \mathbb{N}$ and the same outcome as program $p \in \mathbb{P}$ in all subsequent periods $t > T$. An *ultimately constant* program $y_T c \in \mathbb{P}$ yields outcomes $\{y_1, y_2, \ldots, y_T\} \in X^T$ in the first T periods and the same outcome $x_t = c$ for all $t > T$, $c \in X$. A *constant* program that yields the same outcome $c \in X$ in all periods is denoted by $\boldsymbol{c} \in \mathbb{P}$.

A decision maker has a *preference relation* $\succcurlyeq$ on $\mathbb{P}$. As usual, the symmetric part of $\succcurlyeq$ is denoted by $\sim$ and the asymmetric part of $\succcurlyeq$ is denoted by $\succ$. The preference relation $\succcurlyeq$ is represented by a function $U : \mathbb{P} \to \mathbb{R}$ if $p \succcurlyeq q$ implies $U(p) \geq U(q)$ and vice versa for all $p, q \in \mathbb{P}$. We consider utility function (1).

$$U\left(\{x_t\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1} \left[\sum_{\tau=1}^{T} \beta^{\tau-1} u\left(x_{(t-1)\cdot T+\tau}\right)\right]. \qquad (1)$$

In formula (1), a standard utility function $u : X \to \mathbb{R}$ is continuous, bounded, non-constant on X and determined up to an increasing linear transformation. Discount factors $\delta \in (0, 1)$ and $\beta > 0$ are unique and $T \in \mathbb{N}$ denotes the number of time periods in the short term.

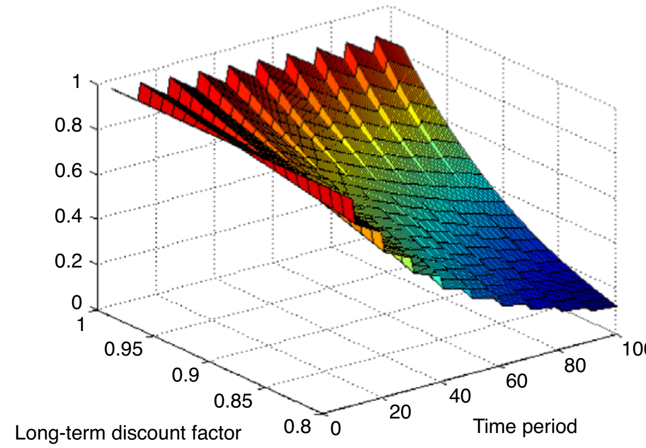

Fig. 1. One unit of utility received in period $t \in \{1, 2, \ldots, 100\}$ evaluated according to (1) for different values of the long-term discount factor $\delta \in [0.8, 0.99]$ and a fixed short-term discount factor $\beta = 0.99$ as well as a fixed length of the short term period $T = 10$.

If $T = 1$ then utility function (1) becomes a conventional discounted utility with one constant discount factor: $U\left(\{x_t\}_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1} u\left(x_t\right)$. Thus, we can interpret discount factor $\delta \in (0, 1)$ as a standard (long-term) discount factor. According to formula (1), utility of outcomes in the first $T \in \mathbb{N}$ periods is $u\left(x_1\right) + \beta \cdot u\left(x_2\right) + \cdots + \beta^{T-1} \cdot u\left(x_T\right)$. Thus, we can interpret discount factor $\beta > 0$ as a short-term discount factor.

If $\delta = \beta^T$ then the short-term discount factor is consistent with the long-term discount factor and a decision maker behaves as if maximizing a standard discounted utility (with one constant discount factor). If $\delta > \beta^T$ then a decision maker exhibits greater impatience for the immediate outcomes in the short term (*i.e.*, he or she is more patient in the long term). Finally, if $\delta < \beta^T$ then a decision maker is more patient in the short term. In fact, model (1) allows for the possibility that a decision maker does not discount outcomes within the first T periods ($\beta = 1$).

Fig. 1 illustrates model (1). Fig. 1 plots the present value of one unit of utility received in period $t \in \{1, 2, \ldots, 100\}$ when it is evaluated by formula (1). We fixed the short-term discount factor $\beta = 0.99$ and the length of the short term period $T = 10$ but allowed for a varying long-term discount factor in the range $[0.8, 0.99]$. When the long-term discount factor is $\delta^* = 0.99^{10} \cong 0.904$ then the future units of utility are evaluated by standard discounted utility with a constant discount factor $\beta = 0.99$. When the long-term discount factor is greater than δ^* a decision maker exhibits greater impatience in the short term (at any point in time the slope of the surface in Fig. 1 is steeper than the slope of its asymptotic trend). When the long-term discount factor is smaller than δ^* a decision maker exhibits greater impatience in the long term (the slope of the asymptotic trend of the surface in Fig. 1 is steeper than the slope of the surface at any point in time).

2. Behavioral characterization

Our behavioral characterization of model (1) is similar to the axiomatization of discounted utility by Bleichrodt et al. (2008).

Axiom 1 (*Completeness*)**.** For all $p, q \in \mathbb{P}$ either $p \succcurlyeq q$ or $q \succcurlyeq p$ (or both).

Axiom 2 (*Transitivity*)**.** For all $p, q, r \in \mathbb{P}$ if $p \succcurlyeq q$ and $q \succcurlyeq r$ then $p \succcurlyeq r$.