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Similarity-based mistakes in choice☆

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ABSTRACT

We characterize the following choice procedure. The decision maker is endowed with two binary relations over alternatives, a preference and a *similarity*. In every choice problem she includes in her choice set all alternatives which are similar to the best feasible alternative. Hence she can, by mistake, choose an inferior option because it is similar to the best. We characterize this boundedly rational behavior by suitably weakening the rationalizability axiom of Arrow (1959). We also characterize a variation where the decision maker chooses alternatives on the basis of their similarities to attractive yet infeasible options. We show that similarity-based mistakes of either kind lead to cyclical behavior. Finally, we reinterpret our procedure as a method for choosing a bundle given a set of individual items, in which the decision maker combines the best feasible item with those that complement it.

1. Introduction

We study mistakes in choice that arise due to similarities between alternatives. In our model the decision maker identifies her most preferred alternative among those that are feasible. However she may mistakenly select an inferior option on the basis of its similarity to the best. Hence her behavior is summarized by a choice correspondence, identifying in every set as *choosable* (i.e., including in the choice set) the best feasible alternative together with any other alternative that is similar to it.

We model similarity as a binary relation over alternatives which substantiates the statement "x is similar to y" in the sense that x could be mistaken for y. For example, a decision maker may mistake a cereal brand for another because they are shelved next to each other. Likewise the low-budget remake *Transmorphers* may be mistaken for the box-office hit movie *Transformers*. Tversky (1977) asserts that similarity need not be transitive, or symmetric. Accordingly, we only impose reflexivity on similarity, i.e., that every alternative should be similar to itself.

Suppose that among three alternatives x, y and z, the decision maker prefers x to y and y to z, and that zSx, meaning z is similar to x. If the decision maker could mistake z for x as a result of this similarity, her behavior would be $c(\{x, y\}) = \{x\}$, $c(\{y, z\}) = \{y\}$ and $c(\{x, z\}) = c(\{x, y, z\}) = \{x, z\}$. Hence, whenever x and z are available in a choice problem, she could choose the inferior alternative z because of its similarity to the superior alternative x. Note this creates cyclical behavior where x is chosen over y, y is chosen over z, but z could be chosen over x.

We characterize this behavior using a single axiom which weakens the classical rationalizability axiom of Arrow (1959). Suppose that a choice problem shrinks in such a way that some of the alternatives in the original choice set remain feasible. Arrow's axiom says that the new choice set should coincide with the set of original choices which are still feasible. Any other consequence, whether a new alternative is included in the new choice set, or one of the original choices is excluded from the new choice set despite being available, is a choice reversal which is incompatible with rationalizability. Our weakening of Arrow's axiom requires the existence of *some* element in any choice set, whose inheritance by subsets prohibits choice reversal. However we do not impose this property on every element in the choice set.

This weakening characterizes similarity-based mistakes. Furthermore, we show that our axiom allows the possibility of binary cycles. This is evident in the example of the previous paragraph. In fact, we find that our axiom, together with the classical no-binary-cycle condition, is equivalent to Arrow's axiom. Hence any departure from rationalizability caused by similarity-based mistakes must induce cyclical behavior.

We also study a related but different model of mistake-making that admits choice of inferior alternatives which are similar to infeasible alternatives that are better than the best feasible alternative. Consider the choice problem $\{y, z\}$ in the previous

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example. Even though y is superior to z and z is not similar to y, the decision maker could mistake z for the infeasible alternative x and include it in the choice set, resulting in $c(\{y, z\}) = \{y, z\}$. It turns out that our weakening of Arrow's axiom and the classical α condition together characterize this behavior. Hence the two models we study are nested: the second is a specialization of the first. Consequently the behavioral consequence of the second type of mistakes is, as in the original model, the possibility of binary cycles.

Our models of mistakes differ starkly from stochastic choice models where behavior is driven by an underlying utility plus a random error term. First, the data that our similarity-based mistakes generate is deterministic. Second, our distortion of pure preference maximization comes in the form of the addition of a reflexive binary relation, the similarity, rather than an error term. Third, our specific procedural view of mistakes allows multiple interpretations. In the concluding section, we briefly describe a model of choice of bundles instead of individual items. By reinterpreting similarity as complementarity, our first procedure gives a boundedly rational way of choosing a bundle: the decision maker first identifies her most preferred item (rather than the most preferred bundle) and next joins it with all feasible items which complement it.

Relation to literature: Our choice procedures operate in two stages. First they identify the best feasible alternative. Next they choose all feasible alternatives which are similar to the best or to any infeasible alternative which is better. There is an important contrast, however, between our work and the recent literature on multistage choice procedures. In this literature alternatives are eliminated in every step until a choice is made.[1] Even though the unique survivor of the first stage remains choosable in our two-stage models, some of the alternatives passed over in the first stage may end up in the choice set as well.

Rubinstein (1988) analyzes a procedure in which a decision maker uses similarities to complete her incomplete preference over lotteries. The role for similarity highlighted in our analysis is different, as our decision maker, fully rational in her mental attitudes towards alternatives, is behaviorally irrational because of similarity-based mistakes.

2. Basic concepts

We consider a standard choice environment with a finite set of alternatives X. The set Σ contains all nonempty subsets of X, and any $A \in \Sigma$ is a *choice problem*. We will omit set brackets and denote sets as strings of alternatives whenever convenient. For example we will write xyz and x instead of $\{x, y, z\}$ and $\{x\}$. In the latter case, the context will clarify whether we are referring to the set or the alternative.

A *choice correspondence* is a map $c : \Sigma \to \Sigma$ satisfying $c(A) \subseteq A$ for every choice problem A. We will call members of $c(A)$ the *choosable* alternatives in A. The interpretation is that any element in the choice set $c(A)$ can end up being the choice and definitely no alternative outside $c(A)$ will be the choice. Hence a choice correspondence is capable of indicating indeterminacy in behavior.

A choice correspondence c is *rationalizable* if there exists a complete and transitive binary relation $\succsim$ on X such that $c(A) = \{x \in A : x \succsim y \text{for all} y \in A\}$. It is well-known that rationalizability is equivalent to the following form of the weak axiom of revealed preference due to Arrow (1959).[2]

Arrow's axiom: For every A and every $B \subset A$, if $B \cap c(A) \neq \varnothing$, then $c(B) = c(A) \cap B$.

Arrow's axiom requires the following consistency of choices. In any small choice problem containing some of the choosable alternatives of a larger choice problem, (i) no new alternative should become choosable, and (ii) no feasible alternative that was originally choosable should drop out of the choice set.

3. Similarity-based mistakes

In this section and the next we will be interested in two procedures for making choices which exhibit mistake-making on the basis of similarities between alternatives. Throughout choices will be determined by two binary relations. The first is a preference R, which we take to be a linear order. We will denote by $\max(A, R)$ the unique maximal element of choice problem A according to R. The second is a similarity S. The next definition clarifies what we mean by similarity.

Definition 1. A *similarity* is a reflexive binary relation on X.

If S is a similarity, the statement xSy indicates that x is similar to y in the sense that, for whatever reason, be it appearance, location, psychological factors, etc., x could be mistakenly chosen instead of y. Similarity need not be transitive, or symmetric. However every alternative is similar to itself. Similarity may or may not be preference-related. We give some examples to illustrate.

Example 1. For some utility representation u of the decision maker's preferences and for some number $\varepsilon > 0$, say x is similar to y if $|u(x) - u(y)| \leq \varepsilon$.

Example 2. For some welfare-irrelevant linear order L which lists the alternatives, say x is similar to y if for any third alternative z neither [xLz and zLy] nor [yLz and zLx]. In this case alternatives are similar if they are adjacent in L.

Example 3. Similarity could be asymmetric if, for example, some alternatives could be mistaken for an attractive alternative x^* but not vice versa. In this case if xSy, then $x = y$ or $y = x^*$.

We now introduce the first choice procedure of our interest.

Definition 2. For any linear order R giving a decision maker's preferences and any similarity S, define a choice correspondence $c_{R,S}$ by

$$c_{R,S}(A) = \{x \in A : xS \max(A, R)\} \tag{1}$$

for every choice problem A. Say c has the similarity-based mistakes representation (1) if $c = c_{R,S}$ for some linear order R and similarity S.

If a choice correspondence admits representation (1), it deems all feasible alternatives which are similar to the best feasible alternative as choosable. Note that since the similarity relation S is reflexive the best feasible alternative is always in the choice: $\max(A, R) \in c_{R,S}(A)$.

We follow with two remarks on $c_{R,S}$.

Remark 1 (*Symmetry Versus Asymmetry of Similarity*)**.** Any similarity that matters in this formulation is that relating a less preferred alternative to a more preferred alternative. Even if a better alternative is similar to an inferior one, this has no bearing on behavior in (1). This leads to the following observation. If $c = c_{R,S}$, then there exist an asymmetric similarity S' and a symmetric similarity S'' such that $c = c_{R,S'} = c_{R,S''}$ as well. To see this, fix a linear order R, a similarity S and suppose $c = c_{R,S}$. Define S' as follows:

[1] See, for example, Manzini and Mariotti (2007), Masatlıoğlu et al. (2012), Dutta and Horan (forthcoming) and Bajraj and Ülkü (forthcoming).

[2] See, for example, Moulin (1985).