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New directions in fuzzy automata

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Abstract

Automata are the prime example of general computational systems over discrete spaces. The incorporation of fuzzy logic into automata theory resulted in fuzzy auotomata which can handle continuous spaces. Moreover, they are able to model uncertainty which is inherent in many applications. Deterministic Finite-state Automata (DFA) have been the architecture, most used in many applications, but, the increasing interest in using fuzzy logic for many new areas necessitates that the formalism of fuzzy automata be more developed and better established to fulfill implementational requirements in a well-defined manner. This need is due to the fact that despite the long history of fuzzy automata and lots of research being done on that, there are still some issues which have not been well-established and issues which need some kind of revision. In particular, we focus on membership assignment, output mapping, multi-membership resolution, and the concept of acceptance for fuzzy automata. We develop a new general definition for fuzzy automata, and based on that, develop well-defined and application-driven methodologies to establish a better ground for fuzzy automata and pave the way for forthcoming applications.

Keywords: Automata theory; General fuzzy automata; Membership assignment; Zero-weight transition; Multi-membership resolution; Output mapping; Acceptance; Conditional acceptance

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1. Introduction

Automata have a long history both in theory and application [1–7]. Automata are the prime example of general computational systems over discrete spaces [8]. Among the conventional spectrum of automata (i.e. Deterministic Finite-state Automata (DFA), Non-deterministic Finite-state Automata (NFA), Probabilistic (stochastic) Automata (PA), and Fuzzy-Finite-state Automata (FFA)), DFA have been the most applied automata to different areas [9–11]. See [12] for more applications. DFA have been shown to be an appropriate tool for modeling systems and applications which can be realized as a finite set of states (including some final states) and transitions between them depending on some input strings [1] e.g. all logic circuits from a simple AND gate to the control unit of a supercomputer. In this paper, we are focussing on Fuzzy Finite-state Automata (FFA), which incorporate fuzziness into the internal state representation and output of these computational systems. Fuzzy automata not only provide a systematic approach to handle uncertainty in such systems, but also are able to handle continuous spaces [15].

There is an increasing interest in using fuzzy logic in many new areas. Fuzzy logic [16] is a very efficient method for handling imprecision which is an intrinsic property of many systems [17]. It provides a nice systematic approach to incorporating approximate reasoning into such systems (in the way humans do) [18,19]. Moreover, fuzzy implementations of many applications are not only cheaper and faster but also make them more understandable for operators and end-users of the systems [20–27,17,28–31].

Fuzzy automata and their counterparts fuzzy grammars, combine the capabilities of automata and language theory with fuzzy logic in an elegant way [32–35]. They have been shown to be very useful for areas which are well-known to be handled by discrete mathematics and probabilistic approaches, e.g. structural matching methods [36], logical design [37]. In general, fuzzy automata provide an attractive systematic way for generalizing discrete applications [38,39,37,40,41]. Moreover, fuzzy automata are able to create capabilities which are hardly achievable by other tools [42]. On the other hand, the contribution of FFA to neural networks (more specifically recurrent NNs) has been considerable, and dynamical fuzzy systems are getting more and more popular and useful [43–46]. It seems that the demand for using FFA will increase considerably in coming years.

In spite of the long history and lots of research being done on fuzzy automata, it still seems that there are some issues which have not been well-established and issues

[1] It is noticeable that our approach is based on the key concept of *state* and the behavior of the automaton is *state-determined*. Hence, an automaton in the scope of our approach is a *discrete-time*, *discrete-state-space*, and *state-determined* machine. But, there is a more general approach where the current state and inputs (and consequently the next state) are not necessarily well-defined [8]. Instead of discrete and well-defined states and inputs, we have the probability distributions of the states and/or the inputs. This gives rise to the concepts of *hyperstates* and *hyperinputs*. However, for the approach we are taking, these are not applicable. For a more detailed discussion on the role of these concepts in modern system theory and their comparison with conventional theory see [8,13,14].