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The marginal tariff approach without single-crossing

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ABSTRACT

We study a specific class of one-dimensional monopolistic nonlinear pricing models without the single-crossing condition. In this class we show that the monopolist optimally splits quantities in two groups: low and high demand. The marginal tariff is sufficient to determine the demand curve (or, equivalently, the monopolist can apply the demand profile approach) within each group. However, given the failure of the single-crossing condition, a global incentive compatibility constraint that prevents deviation across demand groups binds. Therefore, the demand profile approach is no longer valid and we have to modify it accordingly to deal with our problem. We give a complete characterization of its solution.

1. Introduction

The literature on nonlinear pricing models usually assumes, when the type is one-dimensional, that the customer's preferences satisfy the single-crossing condition (SCC). In this paper we study a class of models where the SCC is violated. The tariff design must take into account not only the usual local incentive compatibility constraints, but also global binding ones.

We build on the nonlinear pricing model of Laffont et al. (1987) and study a case where the parameters of the linear demand curve (intercept and slope) are positively correlated (in their case, these parameters are independently distributed). Equivalently, we assume that the market size (maximum demand for positive prices) is inversely related to the choke price (the limit price at which the demand vanishes).

The monopolist faces two conflicting incentive problems when solving the tradeoff between rent extraction and efficiency. For low types, the market size is relatively big and, to sell more to customers who value the good most, the monopolist offers price discounts on quantities to prevent their selection from customers who value the good least. For high types, the market size is relatively small, and the monopolist has limited capacity to sell large quantities, reducing the quantity offered to these types. By doing so the monopolist may pool low and high types and incentive compatibility implies that the marginal tariff paid by pooling types should be the same. This procedure leads to the demand profile approach (DPA) and determines the optimal nonlinear tariff under this approach.[1,2]

When the SCC does not hold, we show that the solution obtained by the DPA can be too restrictive. The monopolist can do better by separating customers in two groups: low and high demand groups. For this, he can set an entry fee which determines a cutoff type who demands the lowest quantity in the high demand group and is indifferent between groups. Customers with types higher (lower) than the cutoff type belong to the high (low) demand group. In each group, the monopolist then applies the DPA.

Comparing to the DPA, these mechanisms allow the monopolist to achieve higher by relaxing global incentive compatibility constraints between types in the high and low demand groups. We indeed implement numerical examples and quantify this profit gain according to an exogenous parameter that captures the market size. Our computation shows that the gain may be significant.[3]

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1 Wilson (1997) is a comprehensive book on nonlinear pricing models and on the DPA.

2 Using the parametric-utility approach, Araujo and Moreira (2010) derived the same solution. Rochet (2009) is another related work in a two-dimensional setup.

3 In Fig. 9 we depict the percentage gain over the solution obtained by Araujo and Moreira (2010).

1.1. Related literature

Mussa and Rosen (1978) is the seminal paper on nonlinear pricing on the provision of quality-differentiated goods. Maskin and Riley (1984) investigate the optimality of quantity discount. Goldman et al. (1984) and Brown and Sibley (1986) offer alternative formulations for solving the monopolistic screening problem, which are equivalent to the DPA, fully developed in Wilson (1997).[4]

When the SCC is valid all these approaches are equivalent. Moreover, according to Armstrong (1996), the main results on models that satisfy the SCC are: (i) methods that give the optimal tariff which has convenient properties[5] such as (ii) complete separation of customers: different types buy different quantities; (iii) no distortion on top: the customer with the highest marginal utility from consumption is efficiently served; and (iv) quantity discount: the optimal marginal tariff is decreasing.

In this paper we relax the SCC as in Araujo and Moreira (2010).[6] The principal difference between their approach and ours is the space of feasible mechanisms. When the SCC fails the optimal allocation can be a discontinuous function of the type. Araujo and Moreira (2010) impose that the type that chooses the limit quantities at a discontinuity is also indifferent to all quantities inbetween these limits. Equivalently, they assume that the feasible allocations are convex-valued correspondences at the discontinuities. As we will show, this restriction in the space of feasible mechanisms is equivalent of using the DPA.

We do not impose this restriction on the space of feasible mechanisms and we are then able to extend the DPA in our framework where the SCC fails. In particular, we show that the partition demand profile approach which consists in separating demand groups can strictly improve the payoff the monopolist under the DPA.

The results of our model where the SCC fails are: (i) the DPA may be suboptimal and we propose an extension of it (the "partition" DPA) that can enhance the monopolist's profit, and combines the DPA within groups of customers with an endogenous division between high and low demand groups; (ii) complete separation of customers may not be possible: there may exist discrete and continuous pooling at the optimal nonlinear pricing; (iii) the strongest type is not necessarily the highest type and may not be served efficiently; and (iv) quantity discounts within each group of customers can still be optimal. However, 'entry fees' can also be part of the tariff structure in order to separate the demand groups. Moreover, the marginal tariff may jump upwards.[7] These features tells us that some interesting properties may arise when the SCC is violated.[8]

4 In a survey on multidimensional screening models, Rochet and Stole (2003) make the distinction between the DPA and the parametric-utility approach.

5 Armstrong (1996) was concerned with how these properties extend to the multiproduct monopoly problem.

6 Nahata et al. (2008) also study screening problems without the SCC. However, in their framework types are finite and they use graph theory to characterize the binding incentive compatibility constraints.

7 These jumps are depicted in Fig. 7. Notice that they are related to the pooling levels q_ℓ and q_h in the quantity assignment function $q(\cdot)$. We also refer the reader to the figures depicted in Wilson (1997, pp. 180–181).

8 There is an interesting link between the failure of the SCC treated in this paper and the family of rotations studied in Johnson and Myatt (2006). They show that if the demand curves faced by the monopolist for different types of customers are ordered according to a family of rotations, there is enough structure on the demand side to provide more specific results. However, they do not consider private information on the customers' types. It is out of the scope of this paper to extend their results when there is asymmetric information, although in our specific framework types are clearly ordered according to a family of rotations. Specific example to make the connection between their class of demand with rotation structure and our guiding example.

1.2. Outline of the paper

In Section 2 we describe our nonlinear pricing model without the SCC. In Section 3 we present the DPA, show how to extend it when the SCC is not valid and to characterize the solution under the partition DPA. In Section 4 we compare the solution derived from the DPA and from the partition DPA. Section 5 concludes. All proofs are relegated to the Appendices.

2. Model

The monopolist and the customer contract on a pair (q, t), with quantity $q \in Q \subseteq \mathbb{R}_+$ and tariff $t \in \mathbb{R}$. The monopolist's profit is $t - C(q)$, where $C(q)$ is the cost function. The customer's preference is represented by $v(q, \theta) - t$, where $\theta \in \Theta = [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}$ is his private information (type). The monopolist has a prior distribution over Θ given by cumulative distribution $F(\theta)$ with a continuous positive density $f(\theta)$. We assume that $C(q)$ is once and $v(q, \theta)$ is three times differentiable.

By the '*Revelation Principle*' the monopolist's problem can be stated as choosing a pair $(q, t) : \Theta \to \mathbb{R}_+ \times \mathbb{R}$ that solves

$$\max_{q(\cdot), t(\cdot)} \int_\Theta [t(\theta) - C(q(\theta))] f(\theta) d\theta, \qquad (\Pi)$$

subject to the *individual-rationality* constraints

$$v(q(\theta), \theta) - t(\theta) \geq 0, \quad \forall \theta \in \Theta, \qquad (\text{IR})$$

and the *incentive compatibility* constraints

$$v(q(\theta), \theta) - t(\theta) \geq v(q(\theta'), \theta) - t(\theta'), \quad \forall \theta, \theta' \in \Theta. \qquad (\text{IC})$$

We say that a quantity assignment function $q : \Theta \to \mathbb{R}_+$ is implementable if there exists a tariff function $t : \Theta \to \mathbb{R}$ such that the pair (q, t) satisfies the incentive compatibility constraint.

Definition 1 (*Single-Crossing Condition*)**.** Consider a utility function $v \in C^2$. We say that v satisfies the single-crossing condition (SCC) or Spence–Mirrlees condition (SMC) when either

$$\forall (q, \theta) \quad \text{in } Q \times \Theta : v_{q\theta} > 0, \qquad (CS_+)$$

or

$$\forall (q, \theta) \quad \text{in } Q \times \Theta : v_{q\theta} < 0. \qquad (CS_-)$$

The SCC allows a simple characterization of implementability. Under (CS_+) (resp. CS_-), quantity assignment function $q(\cdot)$ is increasing (resp. decreasing) if and only if it is implementable.[9]

2.1. Relaxing the single-crossing condition

Following Araujo and Moreira (2010), we extend the SCC by assuming the existence of a decreasing curve q_0 that divides the (θ, q)-plane into two regions: $v_{q\theta} > 0$ below q_0 and $v_{q\theta} < 0$ above q_0. With some abuse of notation, we use CS_+ and CS_- to represent these regions, as illustrated in Fig. 1. Formally, we assume that:

A 1. The equation $v_{q\theta}(q, \theta) = 0$ implicitly defines the function $q_0(\cdot)$ such that $v_{q\theta}(q, \theta) > 0$ when $q < q_0(\theta)$ (CS_+) and $v_{q\theta}(q, \theta) < 0$ when $q > q_0(\theta)$ (CS_-). Moreover, $v_{q\theta^2} < 0$ and $v_{q^2\theta} < 0$ hold on $\mathbb{R}_{++} \times \Theta$.

Propositions 1 and 2 are proven in Araujo and Moreira (2010). They provide the necessary conditions for implementability under assumption A1 and useful properties of all implementable $q(\cdot)$. Let us begin with the necessary local monotonicity conditions.

9 For a proof, see Rochet (1987) or Fudenberg and Tirole (1991, chapter 7).