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Delay, multiplicity, and non-existence of equilibrium in unanimity bargaining games

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ABSTRACT

We consider a class of perfect information bargaining games with unanimity acceptance rule. The proposer and the order of responding players are determined by the state that evolves stochastically over time. The probability distribution of the state in the next period is determined jointly by the current state and the identity of the player who rejected the current proposal. This protocol encompasses a vast number of special cases studied in the literature. We show that subgame perfect equilibria in pure stationary strategies need not exist. When such equilibria do exist, they may exhibit delay. Limit equilibria as the players become infinitely patient need not be unique.

1. Introduction

In his seminal paper, Rubinstein (1982) studies the division of a surplus among two impatient players through a non-cooperative bargaining game. Following this contribution, a rich literature has emerged which extends and generalizes Rubinstein's approach. A number of general results have persistently and recurrently emerged from this literature: in bargaining games where a surplus is divided under unanimity rule, equilibria exist, are efficient, and converge to a weighted Nash bargaining solution in the limit. In this paper, we explore the boundaries of the scope under which these general results are valid.

We consider bargaining games with the following characteristics. There is a finite number of players who need to make a unanimous choice for one particular payoff vector within a full-dimensional set of feasible payoffs. The game is set in discrete time and in each round of the game, one player is selected as the proposer. His role is to suggest one particular feasible payoff vector. The other players then sequentially accept or reject this proposal in some fixed order. If all players agree to the proposal, the game ends and the agreed upon payoffs are realized. As soon as one of the players rejects the current proposal, the game proceeds to the next round. We assume an exogenous breakdown of the negotiation to occur after each disagreement with probability $1-\delta$. Time-discounting and the possibility of an exogenous breakdown are largely interchangeable interpretations of δ. The term "bargaining friction" can be used to capture both of them. The importance of the bargaining friction lies in the fact that it creates an incentive to come to an agreement sooner rather than later.

In order to complete the description of a unanimity bargaining game, one has to specify a rule which determines which player is the proposer in what round. We will refer to this rule as the protocol in the sequel. Rubinstein (1982) studies a game with only two players who simply take turns in making proposals, the alternating offer protocol. Rubinstein finds a unique subgame perfect equilibrium. The equilibrium strategies happen to be stationary.

It is well-known that the uniqueness of subgame perfect equilibrium breaks down in unanimity bargaining games with more than two players. With regard to those games, the literature focuses on subgame perfect equilibrium in pure stationary strategies (SSPE), which allow sharp predictions of the equilibrium payoffs at least when the discount factor is sufficiently close to one. Arguably the most obvious generalization of Rubinstein's alternating offer protocol to the case with more than two players is the rotating protocol, under which players become proposers in ascending order, and the first player proposes again after the last player. One alternative proper generalization of the alternating offers protocol is the rejector-proposes protocol introduced in Selten (1981) in a coalitional bargaining set-up. Under that rule, the

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first player to reject the current proposal becomes the next proposer. The rejector-proposes protocol is an example of an endogenous protocol in which the actions taken by the players throughout the game have an influence on the proposer selection. One important example of a protocol which is not a proper generalization of Rubinstein's alternating offers protocol is the time-invariant probability protocol which consists of an exogenously given probability distribution from which the proposer is drawn in each round.

The literature on unanimity bargaining games has established some results that are generally valid no matter which of these protocols is assumed.

1. An SSPE exists.
2. Every SSPE has no delay.
3. Every SSPE has efficient proposals.
4. There is a unique limit equilibrium as δ approaches one.
5. All limit equilibrium proposals are equal to a weighted Nash bargaining solution.

The aforementioned results follow from Britz et al. (2010) for exogenous protocols and from Britz et al. (2014) for endogenous protocols. They extend earlier work for more specific protocols by Binmore et al. (1986), Hart and Mas-Colell (1996), Kultti and Vartiainen (2010), Laruelle and Valenciano (2008), and Miyakawa (2008).

The weights in the Nash bargaining solution corresponding to the limit equilibrium proposals depend on the distribution of bargaining power inherent in the protocol. Kultti and Vartiainen (2010) show that the weights are all equal to each other under the rotating protocol. Miyakawa (2008) and Laruelle and Valenciano (2008) study the time-invariant probability protocol. In this case, the vector of bargaining weights is given by the time-invariant probability distribution. Britz et al. (2010) study a protocol where the selection of the proposer is described by a Markov process. That is, there are n probability distributions on the n players. The identity of the proposer in the current round determines which of the probability distributions is used to draw the proposer in the following round. This Markovian protocol is both a generalization of the time-invariant probability protocol and the rotating protocol. Then, the vector of bargaining weights is given by the stationary distribution of the Markov process.

Britz et al. (2014) study a class of endogenous protocols, thereby covering the rejector-proposes protocol. They consider protocols which consist of n probability distributions on the n players. The identity of the player who rejects the current proposal determines which of those probability distributions will be used to draw the following proposer. The vector of bargaining weights is shown to be proportional to the vector of probabilities with which the players propose after their own rejections.

In this paper, we present a class of unanimity bargaining games that allows for a rich family of bargaining protocols. This is achieved by introducing a finite set of states and for each state, conditional on the identity of the rejecting player, a vector of transition probabilities to the new states. The state determines the identity of the proposer and the order of the responses. It is easily verified that all the aforementioned protocols follow as special cases. The modeling approach is closely related to the one of Merlo and Wilson (1995). We are more general in allowing for endogenous protocols, i.e. the vector of transition probabilities may depend on the identity of the rejecting player. Motivated by our desire to study the effects of the bargaining protocol itself, we are less general in not allowing for the set of feasible payoffs to depend on the state.

We demonstrate that the results of the bargaining literature as enumerated above do not generalize, even when we require the set of feasible payoffs to correspond to the division of a unit surplus. We construct an example with three players and three states. A player is the proposer in his own state and the states corresponding to Players 2 and 3 are absorbing. Once Player 2 or Player 3 is selected as the proposer, he will remain the proposer forever. In state 1, the protocol follows the rejector-proposes protocol after rejections by Players 2 or 3. In the example, any SSPE predicts delay. SSPE proposals need not be efficient. In the example there is a continuum of SSPEs. Since the example is valid for arbitrarily high values of δ, it is then used to show that limit equilibria are not unique, that there are two accumulation points of limit equilibrium utilities, and that limit equilibrium proposals may not be equal to each other. We show that the example is robust to perturbations of the transition probabilities.

The main intuition behind the example is that Players 2 and 3 capture the entire surplus in their own state. Since the protocol is of the rejector-proposes type in state 1, an SSPE with immediate acceptance requires Player 1 to offer both of them at least δ. Since the total surplus is equal to one, this is clearly infeasible when δ is above one half. All SSPEs are therefore such that the offer by Player 1 in state 1 is rejected by one of the other players.

Yildiz (2003) studies the role of optimism in explaining bargaining delay. He has an example which is similar to ours in the sense that each of the responding players has a continuation payoff of δ, so that immediate agreement is impossible when δ exceeds one half. In the example by Yildiz (2003), however, this result is driven by the fact that due to optimism each player believes that he will become the proposer in the next round with probability one. These beliefs are incompatible and therefore at least one player is wrong about the protocol. In contrast, the delay in our example is derived in a set-up where the bargaining protocol is common knowledge among the players.

Finally, we modify our leading example in a simple way. In state 1, we use the rejector-proposes protocol for Players 2 and 3 with probability one half and assume that Player 1 remains the proposer with the complementary probability. We demonstrate that now both SSPEs with immediate agreement and SSPEs with delay fail to exist. Herings and Predtetchinski (2015) have shown for the time-invariant probability protocol that an SSPE exists even when the set of feasible payoffs is non-convex. It follows that variations in the protocol are more problematic for fundamental properties like existence and efficiency of an SSPE than variations in the set of feasible payoffs.

Our results complement some of the examples of equilibrium delay and non-existence found in the literature. An example of an SSPE exhibiting delay has been given in Chatterjee et al. (1993) in the context of coalitional bargaining. Unlike the unanimity bargaining games considered here, in coalitional bargaining games a proposing player may choose to make an offer to a subset of the players. The approval of the proposal by all players in the chosen coalition is then sufficient for the proposal to pass. Also in a coalitional bargaining context, Bloch (1996) shows that SSPEs need not exist. Merlo and Wilson (1995) show that an SSPE may exhibit delay if the size of the cake changes stochastically over time. Jéhiel and Moldovanu (1995) show that delay can arise due to externalities. In addition to these examples, where delay arises in a complete and perfect information framework, there is a literature on bargaining delays when the parties are asymmetrically informed, see for instance the review by Ausubel et al. (2002).

The plan of the paper is as follows. We start by formally describing a class of unanimity bargaining games in Section 2. Section 3 summarizes the results in the literature regarding existence, immediate agreement, efficiency, and limit equilibria. Section 4 presents the example where an SSPE predicts delay, inefficiency, and non-uniqueness of the limit equilibrium. Section 5 shows the example to be robust to perturbations in the transition probabilities. We show in Section 6 that an SSPE may even fail to exist all together and Section 7 show this example to be robust to perturbations in the transition probabilities. Section 8 discusses the extension of the analysis to coalitional bargaining and Section 9 concludes.