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Editorial

Introduction to financial frictions and debt constraints

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ABSTRACT

This is an introduction to the special section on financial frictions and debt constraints.

Debt, portfolio and solvency constraints are among the most studied financial frictions both in general equilibrium theory and dynamic macroeconomics. Still, many issues remain open, and this special section collects five papers representative of frontier research in both areas. Both existence and uniqueness of competitive equilibria and constrained efficiency characterization are tackled in the two first articles of the section in two different Markovian exchange economies with sequentially complete markets inducing imperfect risk-sharing due to lack of commitment. Ramsey models with heterogeneous agents and (liberal) borrowing constraints are the object of the third article while the two last are motivated by the recent European sovereign debt crisis and international financial crisis and focus on the role of public debt constraints and financial frictions respectively in the emergence of macroeconomic instability and bubbles.

Solvency constraints in infinite horizon Markovian exchange economies with complete markets

A fundamental line of research concerns the role of debt and solvency constraints under uncertainty. The interaction between uncertainty (think of idiosyncratic shocks to fix the ideas) and solvency constraints suggests a large set of questions ranging from existence and uniqueness of competitive equilibria to efficiency concepts, and a large set of possible environments to tackle these questions. A seminal paper in this literature is Kehoe and Levine (1993). One of the numerous merits of this paper is to present an Arrow–Debreu-like theory in a benchmark environment, that is infinite horizon exchange economies with complete and common information, complete contingent claims markets but limited commitment to repay debts. A key ingredient of this theory is the so-called **participation constraints** restricting allocations to be self-enforcing relative to autarkic reservation utilities. Default leads not only to seizing individuals' assets but also, and crucially, to their permanent exclusion from future trading. Uncertainty is idiosyncratic, typically modeled through a Markovian process over a finite number of states, agents have the same von Neumann–Morgenstern utility function and same discounting rate, they may only differ in their per period endowment which is random. Since information is complete, individuals cannot enter into a contract in which they would have an incentive to default in some state. Partial insurance is therefore a natural outcome in this framework. Kehoe and Levine provide the corresponding (constrained and unconstrained) efficiency analysis and welfare theorems.

Alvarez and Jermann (2000) have significantly revisited the equilibrium concepts in the Kehoe–Levine type of economies by shifting the focus from the above mentioned participation constraints on individual consumption sets to the **endogenous** solvency constraints implied by the latter, allowing for an

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illuminating economic analysis in terms of asset prices and a consistent new decentralization procedure. Precisely, Alvarez and Jermann show that Kehoe–Levine's participation constraints do imply endogenous solvency constraints, which reduce effectively risk sharing as agents with low income can only borrow up to the level they can pay back in the future. More importantly, these solvency constraints lead the authors to define a new equilibrium concept according to which agents will make sure that their wealth is not too small to avoid default and reverting to autarky while enforcing at the same time as much risk sharing as possible (this is the so-called *not too tight debt constraints* put forward in this seminal paper).

Interestingly enough, Alvarez and Jermann are able with this new focus to formulate (constrained) efficiency in terms of asset prices, rather than relying on subjective preferences or evaluations of risk. In particular, they find that **high implied interest rates**, that is when the present value of endowments under the Arrow–Debreu price process is finite, are sufficient for constrained efficiency. The connection between high interest rates and constrained efficiency is somehow intuitive in this environment. In the steady state, as the marginal rate of substitution coincides with gross interest rate, reducing current consumption of an individual for equal compensation in the following period is welfare increasing when the interest rate is strictly negative; since this reallocation can be repeated over the infinite horizon, one immediately gets the failure of constrained efficiency under low interest rates. Things are much more involved out of the steady state and under uncertainty. In particular, the necessity of high implied interest rates for constrained efficiency is far less obvious. In Alvarez and Jermann, necessity is obtained in the pure tradition of Kehoe and Levine's proof of the second welfare theorem in the same environment but as in the latter additional conditions are needed.[1]

In a subsequent fundamental contribution, Bloise and Reichlin (2011) observe that this kind of equivalence between high implied interest rates and constrained efficiency in the Kehoe–Levine environment is not general. Indeed, high implied interest rates may not be necessary for constrained efficiency in an economy with non-stationary allocations whereas stationarity is assumed in Alvarez and Jermann's paper. Inspired by the work of Cass (1972) on stochastic overlapping-generations models, Bloise and Reichlin come to an alternative characterization of constrained efficiency in terms of **uniform gains from trade**, that is on the existence of feasible welfare improvements thanks to trade (or risk-sharing), even though a fraction of aggregate endowment is destroyed when departing from autarchy. Precisely, taking the standard general equilibrium approach, they prove that under uniform gains to trade, the support by a linear functional is a necessary and sufficient condition for constrained efficiency for the set of allocations that are uniformly bounded away from zero.

Martins da Rocha and Vailakis (2015), in this special section, elaborate on Bloise and Reichlin's work to provide a major result, relaxing, among others, the assumption of uniform gains from trade in the characterization of constrained efficiency. They proceed in two steps. First, they prove that high implied interest rates are necessary and sufficient for constrained efficiency under uniform gains from trade. With respect to Bloise and Reichlin (2011), the latter result is established without restricting the set of allocations to be bounded away from zero, and more importantly, the sufficiency proof builds on a new decentralization procedure which does not require uniform gains from trade indeed.

In a second step, Martins da Rocha and Vailakis construct perturbed economies: these are simple extensions of the traditional environment where a physical and sizable asset is introduced (physical asset means it is in positive net supply as in Kehoe and Levine). It can be readily shown that if the dividend process of the physical asset is large enough with respect to the private aggregate endowment process, the property of uniform gains to trade is automatically satisfied. In their way to establishing their complete characterization, Martins da Rocha and Vailakis show that a constrained efficient allocation can be obtained as the limit of allocations corresponding to the perturbed economies described above, and therefore exhibiting high implied interest rates. The outcome of the first step then allows to conclude the characterization argument. In addition to closely comparing their characterization to Bloise and Reichlin's, Martins da Rocha and Vailakis provide an illuminating example (on a standard stationary Markovian economy) of the practical interest of their approach and its high operational value.

Another important framework for the analysis of imperfect risk-sharing because of lack of commitment despite sequentially complete markets is explored in this special section. Bloise and Citanna (2015) study the existence and uniqueness of equilibrium in the Kehoe–Levine environment with a major difference with respect to the literature outlined just above: as a fraction of endowment is pledgeable (collateral constraints), which is not systematically the case in the latter literature, no further punishment mechanism is considered. Accordingly, there is no exclusion from trade upon default and the seizure of the collateral by lenders is the only loss an agent faces for his default. A recent exploration into this class of models is due to Gottardi and Kubler (2015). As interestingly pointed out by the latter authors, the level of the borrowing (collateral) constraint is **endogenously** determined in equilibrium by the agents' limited commitment problem like in the Kehoe and Levine environment and in sharp contrast to the typical exogenous treatment of liquidity constraints in the traditional literature of the permanent income hypothesis. Gottardi and Kubler (2015) provide new results on existence and uniqueness of competitive equilibria in this framework with an accurate account of the specific technical problems encountered with respect to the Kehoe–Levine setup.[2] Gottardi and Kubler also deliver a partial efficiency analysis. Precisely, Gottardi and Kubler give some sufficient conditions for competitive equilibria to be fully Pareto efficient, that is for the amount of available collateral to be sufficiently large that the collateral constraints are not binding. In this special section, Bloise and Citanna (2015) provide an illuminating exposition on the existence and especially the uniqueness of equilibrium issue in the same environment as Gottardi and Kubler, which not only allows them to completely solve the latter issue, but also to identify some highly useful and broad methodological clues.

A major achievement of Bloise and Citanna's paper is the complete proof of uniqueness provided, in contrast to Gottardi and Kubler. Both papers use the gross-substitution hypothesis on preferences, an essential ingredient for the investigation of the existence and uniqueness of competitive equilibria in this environment and in many related dynamic models as exemplified

[1] Precisely, Kehoe and Levine (1993) prove that as long as induced Arrow–Debreu prices are strictly positive, high implied interest rates hold at equilibrium. This happens under sufficiently productive assets in positive net supply in Kehoe and Levine. Alvarez and Jermann set an alternative condition in terms of (partial) gains from risk-sharing.

[2] Because the equilibrium concept associated is not a standard Arrow–Debreu equilibrium, it is not possible to derive equilibrium allocations as the solution to a planner's problem as in Kehoe and Levine (1993) and the authors have to deal with technical problems similar to those encountered in the literature of incomplete markets, as in Kubler and Schmedders (2003). See Gottardi and Kubler (2015) for more details.