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Uniqueness of competitive equilibrium with solvency constraints under gross-substitution[☆]

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ABSTRACT

Under a gross substitution assumption, we prove existence and uniqueness of competitive equilibrium for an infinite-horizon exchange economy with limited commitment and complete financial markets. Risk-sharing is limited as only a part of the private endowment can be used as collateral to secure debt. The unique equilibrium is Markovian with respect to a minimal state space consisting of exogenous shocks and Negishi's welfare weights. We represent equilibrium dynamics via a monotone operator acting on entire wealth distribution functions. We construct a fixed point of this operator generating a lower and an upper orbit and proving coincidence of accumulation points.

1. Introduction

We offer an existence and uniqueness proof of competitive equilibrium in an infinite-horizon exchange-economy under solvency constraints. Because of the lack of commitment, risk-sharing is limited even though financial markets are sequentially complete. Indeed, as in Lustig (2000), Chien and Lustig (2010) and Gottardi and Kubler (2015), only a fraction of the private endowment is pledgeable and can serve as collateral to secure debt. Unsecured debt is unsustainable due to the absence of other enforcement mechanisms, as the exclusion from trade upon default (Kehoe and Levine, 1993; Alvarez and Jermann, 2000; Hellwig and Lorenzoni, 2009). Pledgeable resources can be basically regarded as dividends of tradable long-term securities.

Fundamental ingredient for uniqueness, preferences are assumed to display the gross substitution hypothesis (with one physical good, this is implied by a relative risk aversion coefficient not greater than one). Gross substitution has been the focus of attention of several papers on economic dynamics, including Boldrin and Levine (1991), Kehoe et al. (1991), Dana (1993) and Gottardi and Kubler (2015). The uniqueness of competitive equilibrium is well-established when each individual is subject to a single budget constraint or when sequential budget and solvency constraints can be consolidated into an equivalent single intertemporal budget constraint. This reduction is, in general, unfeasible under solvency constraints and the established literature cannot be invoked. To the best of our knowledge, the only other study of uniqueness under solvency constraints is Gottardi and Kubler (2015).

The unique competitive equilibrium is recursive in that the state space defining the transition probabilities is reduced to the pair of exogenous shock and Negishi's welfare weights. This is also the peculiar structure of competitive equilibrium with unsecured debt under permanent exclusion from trade upon default (Kehoe and Levine, 1993; Alvarez and Jermann, 2000). Indeed, because of constrained efficiency, all recursive dynamics are governed there by the evolution of welfare weights, accounting for movements along the constrained efficient Pareto frontiers. In the economy with secured debt, where no constrained efficiency is straightforwardly available, gross substitution imposes a similar discipline at equilibrium.

In a long-standing tradition in the applied literature (see, e.g., Heaton and Lucas, 1996; Zhang, 1997; Krusell and Smith, 1998), a recursive competitive equilibrium state space is given by pairs of exogenous shock and distribution of initial wealths (or portfolios) across individuals. In general, existence of such recursive equilibria fails, even for economies with perfect markets (see Kubler and Schmedders, 2002). In the economy with collateral under complete markets, because of the overall uniqueness implied by gross substitution, Negishi's weights can be injectively

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mapped into distributions of wealth and, thus, competitive equilibrium is recursive also with respect to this alternative state space.

Our proof uses the monotonicity, afforded under gross substitution, of the transition dynamics of welfare weights. In particular, we represent equilibrium dynamics via a monotone operator acting on entire wealth distribution functions. The proof does not apply a fixed point theorem on the standard domain of this operator; rather, it constructs two isotonic sequences of functions using monotonicity and the upper and lower bounds of the domain. It then shows that the limit functions exist and coincide in the domain of the operator. Existence, uniqueness of equilibrium, and the Markovian property then follow all at once. Though this method is a common practice when the Contraction Mapping Theorem is not available, we need genuinely innovative arguments for the overall procedure to go through.

Gottardi and Kubler (2015) present an incomplete proof of uniqueness of competitive equilibrium. Their approach consists in applying the logic of Kehoe et al. (1991) to show directly the uniqueness of competitive equilibrium via a state-by-state minimum operation over welfare weights. As in Gottardi and Kubler (2015), we also describe equilibria via adjustments of the Negishi's weights and use the gross substitution hypothesis to establish that adjusted weights are unique at each contingency. The advantage of our approach is that we also provide an efficient computational tool, as iterated applications of our operator uniformly converge to the unique fixed point. Furthermore, as the operator is monotone, it may also deliver comparative statics insights, which are deferred to future research.

It is worth remarking that (a slight adaptation of) our method can prove uniqueness of competitive equilibrium under gross substitution for arbitrarily predetermined debt limits. Furthermore, as equilibrium is uniquely determined as a fixed point of a monotone operator, we expect monotone readjustments when varying debt limits. In the literature on limited commitment, not-too-tight debt limits are endogenously determined by participation constraints. This introduces a potential non-monotonicity which might destroy the regularity delivered by gross-substitution and induce complicated dynamics at equilibrium (see, for instance, Azariadis and Kaas, 2013). However, under some conditions, the endogenous debt limits exhibit a form of dynamic complementarity: given prices, more permissive debt limits in the future induce more permissive debt limits in the present. In such cases, monotonicity is preserved and, thus, *probably* is the uniqueness of competitive equilibrium under gross-substitution. This understanding conflicts with some recent literature on endogenous credit cycles (Gu et al., 2013).

As a last remark, we observe that our analysis requires non-vanishing pledgeable resources. This rules out the extreme situation in which all private endowments are unpledgeable. In this case, there is always a trivial competitive equilibrium with no trade. However, trade might still occur at equilibrium sustained by speculative bubbles or unbacked public debt. Hellwig and Lorenzoni (2009) show that this equilibrium with trade is equivalent to an equilibrium with unsecured debt when exclusion from future borrowing (but not future lending) is the punishment for default. When the economy is perturbed by introducing an infinite-maturity security paying off some arbitrarily small (though non-negligible) dividends, the equilibrium with unsecured debt in Hellwig and Lorenzoni (2009) coincides with the equilibrium with secured debt in our paper. Thus, by approximation, our analysis also reveals some regularities of the competitive equilibrium studied in Hellwig and Lorenzoni (2009).

The paper is organized as follows. Section 2 introduces the economy and gives some basic definitions. Section 3 defines the competitive equilibrium and provides a characterization in terms of first-order conditions. Section 4 presents the recursive framework of analysis. Section 5 contains the main results. Section 6 offers an extension to economies with multiple goods. The Appendix contains the proofs.

2. Fundamentals

2.1. Time and uncertainty

Time is discrete, and denoted by t in $\mathbb{T} = \{0, 1, 2, \ldots\}$. Uncertainty is represented by a probability space, $(\Omega, \mathcal{F}, \pi)$, and a filtration of σ-algebras, $(\mathcal{F}_t)_{t\in\mathbb{T}}$. To simplify, at every t in $\mathbb{T}$, $\mathcal{F}_t$ is generated by a *finite* partition of Ω. In period t in $\mathbb{T}$, for every state of nature ω in Ω, $\mathcal{F}_t(\omega) = \cap\{E_t \in \mathcal{F}_t : \omega \in E_t\}$ defines the prevailing event. Throughout the analysis, we refer to any such events as *contingencies*. In the equivalent event-tree representation, a contingency is a date-event.

The following pieces of notation are used hereafter. Given a topological space V, endowed with its Borel σ-algebra, $L(V)$ defines the space of all V-valued stochastic processes $f : \mathbb{T} \times \Omega \to V$ adapted to the filtration. For any given period t in $\mathbb{T}$, the space of all $\mathcal{F}_t$-measurable maps $f_t : \Omega \to V$ is $L_t(V)$. For a (subset of a) normed space V, we consider the subspace $L_\infty(V)$ of $L(V)$ containing only bounded adapted processes, that is, those satisfying

$$\sup_{t\in\mathbb{T}} \sup_{\omega\in\Omega} \|f_t(\omega)\|_V \text{ is finite.}$$

Whenever the space V is ordered, $L(V)$ inherits the ordering. The term positive is used in the weak sense. Thus, for instance, an element f of $L(\mathbb{R})$ is positive if $f_t(\omega) \geq 0$ for every t in $\mathbb{T}$ and every ω in Ω. The positive cone of $L(\mathbb{R})$ is denoted by $L^+(\mathbb{R})$. The strictly positive cone is $L^{++}(\mathbb{R})$, i.e., $f_t(\omega) > 0$ for every t in $\mathbb{T}$ and every ω in Ω.

Markov (recursive) processes are encompassed as follows in this sequential framework. For a finite state space S, a Markov transition is a map $\Pi : S \to \Delta(S)$. Uncertainty is Markov reducible whenever there exists an adapted process σ in $L(S)$ such that, at every t in $\mathbb{T}$, for every measurable set E of S,

$$\pi(\sigma_{t+1} \in E | \sigma_t = s) = \Pi_s(E).$$

An adapted process f in $L(V)$ is Markov reducible if there exists a map $\vec{f} : S \to V$ such that $f = \vec{f} \circ \sigma$. We shall assume that fundamentals are Markov reducible.

2.2. Agents, preferences, endowments

There is a finite set I of infinitely-lived agents. Their preferences over consumption plans, i.e., over elements of $L^+(\mathbb{R})$, are represented by a time and state separable discounted utility with Bernoulli index $u^i : \mathbb{R}_+ \to \mathbb{R}^*$ and discount factor δ in $(0, 1)$, where $\mathbb{R}^*$ denotes the real line extended so as to include negative infinity. The utility function u^i is bounded from above, smooth, smoothly strictly increasing and smoothly strictly concave with

$$\lim_{x^i\to 0} \nabla u^i\left(x^i\right) = \infty. \tag{I}$$

Furthermore,

$$\lim_{x^i\to 0} \nabla u^i\left(x^i\right) x^i \text{ is finite.} \tag{F}$$

An agent's utility from consumption plan x^i in $L^+(\mathbb{R})$ is

$$U_0^i(x^i) = \mathbb{E}_0 \sum_{t\in\mathbb{T}} \delta^t u^i(x_t^i).$$

It is understood that this value might be negative infinity (and so is whenever $u^i(0) = -\infty$ and consumption vanishes at some contingency).

Each agent is endowed with e^i in $L_\infty^{++}(\mathbb{R})$ units of the single consumption good throughout their life. Each agent i in I is also endowed with a portion of a tree, with payoff y^i in $L_\infty^+(\mathbb{R})$ such that the whole tree $y = \sum_{i\in I} y^i$ lies in $L_\infty^{++}(\mathbb{R})$. Both processes e^i