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Portfolio constraints, differences in beliefs and bubbles☆

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ABSTRACT

I propose an arbitrage-based theory of bubbles in economies with general portfolio constraints and differences in beliefs. I find that, in general, bubbles cannot exist unless the constraints restrict the demand for credit sufficiently to induce low interest rates. Speculation due to heterogeneous beliefs does not cause bubbles. Ruling out bubbles under asymmetric information requires stronger assumptions: the presence of some uninformed agents and mild portfolio restrictions (debt or borrowing constraints), or alternatively, the existence of some impatient and fully informed agents.

1. Introduction

Large increases in asset prices, followed by rapid collapses, are seen as evidence of *bubbles*. A bubble measures the portion of an asset price in excess of its *fundamental value*, calculated typically as some expected discounted present value of its dividends. Bubbles can exist only in the presence of trading constraints or informational restrictions preventing agents from shorting the overpriced assets. The three main types of frictions used in various theories of bubbles are heterogeneous beliefs, asymmetric information, and financial frictions (portfolio constraints).

Heterogeneous beliefs can lead to speculation, and hence can result in overvalued assets and bubbles (Harrison and Kreps, 1978; Morris, 1996; Scheinkman and Xiong, 2003).[1] *Asymmetrically informed* agents might trade in overvalued assets as more informed agents expect to sell them before the crash to less informed agents (the "greater fool" theory) (Allen et al., 1993; Abreu and Brunnermeier, 2003; Doblas-Madrid, 2012). Bubbles due to heterogeneous beliefs or asymmetric information, coupled with short sale constraints, are usually referred to as *speculative bubbles*. Bubbles due solely to (sufficiently severe) *portfolio constraints*, without any asymmetries of information or beliefs, are known as *rational bubbles*.[2] The portfolio constraints can be exogenous (Kocherlakota, 1992; Hugonnier, 2012),[3] or can be endogenized by various financial frictions such as pledgeability limitations, limited enforcement or limited collateral (Kiyotaki and Moore, 2008; Kocherlakota, 2009; Hellwig and Lorenzoni, 2009; Hirano and Yanagawa, 2010; Miao and Wang, 2011; Giglio and Severo, 2012; Farhi and Tirole, 2012; Bidian, 2015; Bidian and Bejan, 2015).

I give a unified, arbitrage-based theory of bubbles, in an environment that simultaneously allows for *heterogeneous beliefs*, *asymmetric information* and a *general class of portfolio constraints*, including borrowing constraints, debt constraints, short sale constraints, margin requirements, etc. I show that a necessary condition for bubbles in positive supply assets is that agents perceive that interest rates (discount rates) are *low*, making the present value of aggregate endowment infinite. Intuitively, bubbles grow on average at the rate of interest rates. With *high* interest rates (finite present value of aggregate endowment), the bubble must become large relative to the aggregate endowment. Optimizing, forward looking agents, will not allow their financial wealth to become too large relative to the present value of their future consumption.

The results suggest that with a consistent definition of bubbles across different class of models, heterogeneous beliefs, and even asymmetric information to a lesser extent, do not cause bubbles. Instead, what creates room for bubbles are sufficiently severe financial frictions, driving down the interest rates by impeding the access of borrowers to credit. Recent empirical work finds support for low interest rates. Geerolf (2013) overturns the findings of Abel et al. (1989), and argues that the return on capital in developed economies is low.[4]

☆ This paper is based on Chapter 2 in Bidian (2011).

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1 For a review, see Xiong (2013). The heterogeneity of beliefs captures the existence of overconfident traders suffering from behavioral biases.

2 For a recent review, see Miao (2014).

3 Similarly, the overlapping generation model of bubbles of Tirole (1985) has built-in constraints, as the old agents have to maintain positive wealth. They cannot borrow against the income of their offsprings.

4 He uses an expanded data set and a more accurate methodology to impute the share of land in national income.

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Such (rational) bubbles driven by financial frictions have important macroeconomic and welfare effects, as they relax the underlying financial frictions. They typically increase the net worth and borrowing capacity of productive entrepreneurs and are expansionary, which seems to be consistent with the recent bubble episodes experience in US (Martin and Ventura, 2012; Bidian, 2014).[5] In particular, when the portfolio constraints are in the form of debt limits, bubbles are entirely equivalent to a relaxation of debt limits and can counteract a credit crunch (Kocherlakota, 2008; Bidian, 2011; Bejan and Bidian, 2014). On the other hand, models of bubbles due to differential information (speculative bubbles) do not shed light on the macroeconomic effects of bubbles. Speculative bubbles simply reduce social welfare, as (speculative) trading is just a negative sum game (Xiong, 2013), making agents' consumption more volatile than their endowments and reducing the sum of their expected utilities.

The paper most closely related to mine is Santos and Woodford (1997). They showed that with *symmetric information*, *homogeneous beliefs* and *borrowing constraints*,[6] bubbles on assets in positive supply require low interest rates.[7] Their theorems do not apply to some of the newer models of rational bubbles cited above, which use different types of portfolio constraints. My results, when restricted to environments without differential information, offer an explanation for the universality of low interest rates in models of rational bubbles.[8]

The absence of bubbles under asymmetric information was also claimed by Tirole (1982), in a model with risk neutral agents and only one asset. However, as pointed out by Kocherlakota (1992), he overlooked the crucial need for portfolio restrictions. Without them, agents can run Ponzi schemes and no equilibrium (and of course, no bubbles) can exist. Yu (1998) allowed for asymmetric information in the framework of Santos and Woodford (1997) with agents subject to borrowing constraints and showed that their theorem on the non-existence of bubbles when agents are impatient and interest rates are high, is still true. Werner (2014) extends the same result of Santos and Woodford (1997) to debt constraints, under symmetric information and homogeneous beliefs. By contrast, the results of this paper applies to economies with a variety of portfolio constraints (including borrowing, debt, or short sale constraints) and heterogeneous beliefs, in addition to asymmetric information. Moreover, only Section 3 makes use of the impatience assumption.

With the general portfolio constraints and differential information considered here, several difficulties arise. First, (limited) arbitrage opportunities can exist in equilibrium. Second, the fundamental theorem of asset pricing, or "martingale-pricing", may not hold. Thus, there might not exist a discount factor which makes the price of the assets equal to the sum of expected discounted value of next period dividends and resale price (this is the case, for example, for short sale constraints). Third, due to the heterogeneity in agents' constraints and information, the notion of discount factors and high (low) interest rates are agent specific.

The starting point is the absence of *unrestricted* arbitrage opportunities, which are arbitrage opportunities that can be added to any feasible trading strategy and scaled up arbitrarily. Equivalently, there cannot be arbitrage opportunities in the *recession* cones of agents' constraints. A Farkas–Stiemke lemma for polyhedral cones[9] establishes the existence of agent specific discount factors (state price densities) that can be used in discounting dividends. Interest rates are high from the point of view of an agent if the present value of aggregate endowment is finite under all agent's discount factors. Equivalently, high interest rates for an agent amount to the agent being able to secure a dividend stream in excess of the aggregate endowment, using only unrestricted trading strategies (in the recession cone of the portfolio constraints). This follows from the duality result in Huang (2000).

Given an arbitrary discount factor derived from the absence of unrestricted arbitrages for a given agent, the price of an asset can be decomposed into three nonnegative components. The first term is the discounted present value of the asset's dividends. The second term in the decomposition represents the value of the *resale option* afforded to the agent by being long one unit of the asset. The term was coined by Scheinkman and Xiong (2003) and captures the excess over what the agent is willing to pay if he cannot trade the asset in the future.[10] Finally, the third component is given by the asymptotic expected discounted value of the asset, and will be referred to as a bubble (under the chosen discount factor), whenever it is nonzero. In discounted terms, the bubble is a martingale. This is the usual definition of a rational bubble found in the literature.[11]

Bubbles can be perceived by some agents, under some discount factors (*ambiguous* bubbles), or by all agents, under all discount factors (*unambiguous* bubbles). Section 3 shows that, by imposing the same form of impatience on agents as Santos and Woodford (1997), bubbles are absent under *any* discount factor associated to an agent with high interest rates (Theorem 3.1). Thus ambiguous or unambiguous bubbles are not possible with high interest rates.

Section 4 drops the impatience assumption. Theorem 4.1 shows that there are no bubbles in assets in positive supply from the point of view of uninformed (having only public information) agents if they perceive that interest rates as high. Intuitively, there is no benefit for the uninformed agents to trade a bubble, since the bubble cannot be sold to a less informed agent. This result rules out only unambiguous bubbles. Two corollaries follow for particular portfolio constraints, without requiring the presence of uninformed agents, but at the cost of additional assumptions. If agents face no *short sales* restrictions and if there is an agent with high interest rates that is unconstrained in a given asset infinitely often, then there exists a discount factor associated to that agent under which the asset is bubble-free. Alternatively, if agents face *debt* or *borrowing* constraints and markets are *complete* from the point of view of a (hypothetical) uninformed agent having high interest rates, then ambiguous bubbles can also be ruled out—there

5 Bubbles can reduce welfare in models with endogenous growth, by lowering the growth rate of the economy (Grossman and Yanagawa, 1993). However, a bubble attached to productive assets (equity) can increase growth, by increasing the return to innovation, despite crowding out the accumulation of (knowledge) capital (Olivier, 2000; Tanaka, 2011). A similar point is made by Miao and Wang (2014), in a different environment.

6 Borrowing constraints impose lower (negative) bounds on an agent's end of period financial wealth.

7 For deterministic economies, the results of Santos and Woodford (1997) were anticipated by Kocherlakota (1992), and later refined by Huang and Werner (2000). They were extended to continuous time environments by Loewenstein and Willard (2000). Montrucchio and Privileggi (2001) show that under mild assumptions on agent's preferences, bubbles cannot exist in a representative agent economy.

8 Santos and Woodford (1997) allow for limited participation and multiple goods. These ingredients do not affect the mechanism through which high interest rates rule out bubbles, and would make the notation daunting, given the other dimensions along which I generalized their framework. The proofs here are simpler despite the general portfolio constraints and differential information.

9 See Huang (2000) or Appendix for a simpler result applying to general closed convex cones.

10 Pascoa et al. (2011) and Araujo et al. (2011) refer to the value of the resale option as the *shadow price* of agent's constraints. It represents the value of all future *services* in relaxing (binding) financial constraints. Cochrane (2002) further interprets the value of the resale option as the *convenience yield* generated by being long one unit of the asset, as holding inventories helps to better smooth demand in the presence of shorting restrictions.

11 It coincides with the definition of Santos and Woodford (1997), since with borrowing constraints the value of the resale option is always zero.