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An auction game model for pool-based electricity markets

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Abstract

A single-period auction game model for analyzing strategic behavior in pool-based electricity markets is introduced in the paper. We study the Nash equilibrium in a pure strategy sense of such games. First an equilibrium existence lemma is proved. Equilibrium characterization under tight capacity constraints is provided. Then it is demonstrated that an auction game does not possess a pure strategy Nash equilibrium under a wide range of market conditions. The paper provides a characterization of equilibrium under weak capacity constraints. We apply the introduced results to analyze market power indices presented in our earlier work and in related reports. Applications to actual market analysis, as well as limitations of the introduced model are provided.

Keywords: Power system; Electricity market; Game theory; Optimization

1. Introduction

Electricity markets are emerging in many parts of the world. The real world market models lie between two extremes—the bilateral model and the pool-based model [1,2]. The micro-features of both bilateral and pool models are being continuously enhanced, debated, and studied [3]. A concern of increasing interest is the development of analytical tools for studying electricity markets [29]. Several approaches are reported in the literature: empirical, experimental [4–6], general competitive market [7,8], and a game-theoretic approach.

This paper focuses on the development of a game-theoretic model. Of particular interest is a characterization of the pure strategy game equilibrium. We study a market auction game for a basic trading period (say, an hour). The unit commitment issue is not considered.

A number of game-theoretic models have been suggested in the literature. The classic Cournot models [9,10], Bertrand models [11], and the recent supply function equilibrium models [12,13], are only a few examples. More examples can be found in [14–18] and references cited therein. A related problem is the optimal bidding issue in electricity markets [19,20].

In a pool-based electricity market, a supplier aims to maximize its profit, but the independent system operator (ISO) always selects least expensive generators to supply power. Therefore a power supplier faces a bi-level optimization problem [21]. This important characteristic of an auction game is recognized in several recent works [22–24].

The game model suggested in this paper follows the aforementioned concept. In contrast to existing models, bid prices, instead of generation output, are assumed to be the strategic variable in the proposed model. We take a case-by-case-study approach, as is often the situation in game-theoretic studies. We also adopt a zonal transmission model, which is widely used by utilities, including NEPOOL [25,26]. We are able to obtain certain analytical, versus computational, results. Notably, we characterized the equilibria of the game under tight and weak capacity constraints. We introduce the concept of quasi-equilibrium for the study of market performance under some market conditions. In a companion paper we present equilibrium analysis results which are based on the assumption that all generators in the market under study have identical cost structures [33]. This assumption is not needed in the results presented here.

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The related work, to the best of our knowledge, is presented in reference [11]. Our results distinguish those of [11] in several aspects. First, we provide equilibrium analysis results which are computable, not just of 'existence' nature as those in [11]. Second, we explicitly model the impact of capacity constraints on market behavior and present market power indices that can be conveniently used in applications.

2. Electricity auction game

Like most of the game models reported in the literature, the proposed model is intended for a single-period auction game. Unit commitment is not considered in this paper. In this section we will first describe the auction and pricing structure, then define the auction game.

2.1. The single-period auction in electricity markets

Let **s** be the bid prices of generators and **P** be the power dispatch vector. The power dispatch and pricing solution is then obtained from the following optimization problem [1]:

$$\min_{\mathbf{P}} \quad \mathbf{s}^T \cdot \mathbf{P} \tag{1.1}$$

$$\text{S.T.} \quad \mathbf{e}^{\mathrm{T}}(\mathbf{P}-\mathbf{L}) = 0, \quad \mathbf{T}(\mathbf{P}-\mathbf{L}) \le \bar{\mathbf{F}}, \quad 0 \le \mathbf{P} \le \bar{\mathbf{P}} \tag{1.2}$$

where $\bar{\mathbf{P}}$ is the high operating limit vector, **e** is a vector with all ones, and **L** is the load vector. The vector $\bar{\mathbf{F}}$ contains certain transmission limits. Eq. (1.2) describe the feasible set of the dispatch problem. The above power dispatch and pricing model is assumed to be loss-less.

In Eq. (1.2), matrix **T** contains the configuration data of a transmission network that is represented by a zonal model [3]. Each zone is subject to an import limit. For example, suppose there are three generators in a zone, the zonal import constraint is represented as follows:

$$-P_1 - P_2 - P_3 + L \le \bar{F}$$

The variations of this zonal model, which can accommodate common transmission constraints [25,26], are being followed in several real world markets (e.g., Australia, Nordic Pool). Zones do not intersect with each other but can be nested, therefore **T** has a simple structure. For example, a small zone consists of generator A, B, and C. This small zone can be included in a larger zone, which consists of generator A, B, C, and D. The information of flows between two zones is not explicit in the model. In many markets where metering is insufficient to support nodal pricing (e.g., New York and New England), using the above zonal transmission model is the only choice for settlement purposes. The elements of the matrix are equal to 0, 1 or −1.

The Lagrangian function of the above optimization problem is as follows:

$$\Gamma = \mathbf{s}^T \cdot \mathbf{P} + \boldsymbol{\lambda}\mathbf{e}^T(\mathbf{P}-\mathbf{L}) + \boldsymbol{\gamma}^T[\mathbf{T}(\mathbf{P}-\mathbf{L}) - \bar{\mathbf{F}}] + \boldsymbol{\alpha}^T(\mathbf{P}-\bar{\mathbf{P}}) - \boldsymbol{\beta}^T\mathbf{P} \tag{2}$$

where the bold Greek characters $\boldsymbol{\lambda}$, $\boldsymbol{\gamma}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ represent Lagrangian multipliers. These Lagrangian multipliers and the dispatch **P** must satisfy the so-called Kuhn-Tucker optimality conditions.

Exactly four alternatives of the auction problem (1) can be seen. They are: (i) the problem is unbounded; (ii) the problem is infeasible; (iii) the problem has a unique optimal solution; and, (iv) the problem has multiple optimal solutions. Under very weak conditions, we will show that the first two alternatives can be excluded from consideration throughout this paper. We are then left with the third and fourth alternatives.

When the auction problem (1) has a unique optimal solution, the spot price of electricity in a zone is equal to the marginal cost of supplying one unit of electricity to that zone. Following the Envelope Theorem [7], the spot prices in zones are computed by differentiating with respect to load as follows:

$$\boldsymbol{\rho} = \frac{\partial \Gamma}{\partial \mathbf{L}} = -\lambda\mathbf{e} - \gamma^T\mathbf{T} \tag{3}$$

If the problem (1) has multiple optimal solutions, it implies that the bid prices of some units are the same and these generators set clearing price. Under such circumstances, these generators are dispatched in proportion to their bid-in quantities. This widely accepted method is important in this paper, and therefore is presented mathematically (for illustration, in the following equation we assume that the bid prices of two generators are the same):

$$P_1 = \frac{\bar{P}_1}{\bar{P}_1 + \bar{P}_2}\hat{L}, \quad P_2 = \frac{\bar{P}_2}{\bar{P}_1 + \bar{P}_2}\hat{L} \tag{4}$$

where $\hat{L}$ is the residual load that the two generators supply (see Fig. 1).

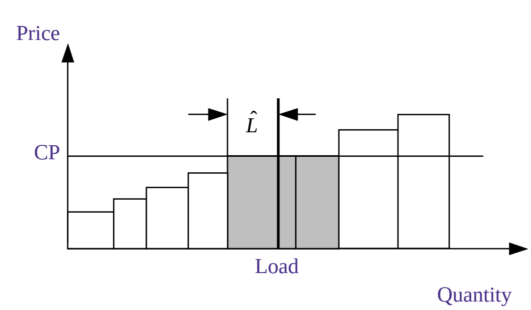

Fig. 1. Residual load when two generators bid the same price (CP: Clearing Price; $\hat{L}$: residual load as defined in the figure). For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.