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Optimal pollution control with distributed delays

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HIGHLIGHTS

- Pollution control model with distributed delays reflecting space/time heterogeneity.
- Analysis of two-dimensional systems of mixed type functional differential equations.
- Full localization of roots of dynamic system with general advance and delay kernels.
- Hopf bifurcation theorem with general advance and delay kernels.
- Sensitivity of dynamics to distributions' parameters.

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ABSTRACT

We present a model of optimal stock pollution control with general distributed delays in the stock accumulation dynamics. Using generic functional forms and a distribution structure covering a wide range of distributions, we solve analytically the complex dynamic system that arises from the introduction of these distributed delays. From a theoretical standpoint, our contribution extends the dynamic optimization literature that focused on single discrete delays and develops an original method to address control problems written as mixed type functional differential equations with general kernels. Our results show the qualitative impact of acknowledging these distributed delays on the optimal pollution paths dynamics. We study analytically the properties of the dynamics and we identify the conditions for the occurrence of limit cycles. This theoretical work contributes to the design of efficient environmental policies in the presence of complex delays.

1. Introduction

Since the seminal contributions of Keeler et al. (1972) and Plourde (1972), partial equilibrium stock pollution control models have been discussed and enriched in various ways with the introduction of uncertainty, multiple pollutants, irreversibility, technological change, etc. However, apart from a few exceptions presented below, this vast literature systematically assumes that the time of emission is tantamount to the time of contamination. This assumption leaves out a crucial aspect of many pollution problems that feature significant delays in the accumulation process. For instance, the contamination of aquifers by leaching nitrates from agricultural sources can occur several decades later (Kim et al., 1993), which in some cases explains why reductions in nitrogen-loaded inputs are not immediately followed by a decrease in downstream water pollution (Grimvall et al., 2000).

From a theoretical point of view, the addition of these delays to the standard optimal stock pollution control framework modifies the properties of the optimal pollution path. Winkler (2011) studies the properties of a model with a discrete delay depending on whether the objective function is separable or not. Using a separable objective function in a model with heterogeneous polluters, Bourgeois and Jayet (2011) show that longer time lags lead to a higher optimal pollution stock at the steady state and that this effect is amplified by asymmetric information. The common feature of these contributions is that they use a single discrete delay, assuming that an emission at time t will reach entirely and systematically the pollution stock at time $t+\tau$. This kind of delay merely translates the dynamic path and leaves its mathematical properties relatively unaffected.

These models with a discrete delay imply nonetheless that the accumulation process is perfectly homogeneous and they ignore

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the possibility of differentiated accumulation velocity of polluting emissions. Site-specific conditions, such as soil heterogeneity in the case of water contamination by nutriments or temperature and pressure variations in the case of greenhouse gases, can cause significant variability in the time frame of pollution. To better capture the intricate lags phenomena, the time of accumulation of these pollutants should in fact be distributed along a time interval following the emission. Such distributed delays cast light on the challenging task of assessing the link between the time and amount of emissions and the time and intensity of the damage they trigger. They raise significant technical difficulties which, contrary to the single discrete delay model, cannot be overcome easily even for a separable objective function. The application of Pontryagin's principle to this model with distributed delays gives rise to a system of optimality conditions that includes at the same time leads and lags, turning the system into Mixed Type Functional Differential Equations (MFDE). In a different field, the model developed by Buonomo and d'Onofrio (2013) to analyze the optimal conduct of politicians when public awareness of their honesty is delayed in time also presents some mathematical similarities to the one we address. Nevertheless we study a general distribution of delays while they focus on two specific delay kernels, which leads to significant differences in the resolution of the problem that are discussed afterwards.

Our aim is to characterize analytically these complex dynamics and the stability conditions of a model using general functional forms. To do so we extend the approach used by Boucekkine et al. (2005, 2010) who apply Pontryagin's approach to vintage capital[1] and we resort to an original method to address the MFDE at stake. Our main contribution to the optimal control literature consists in establishing analytically several properties that enable us to locate the roots of the characteristic equations of this range of models, and thus to characterize the qualitative properties of the dynamics. This is made possible by our use of a general delays kernel which covers a wide range of distribution structures. Consequently, our results enrich economic theory by showing that when a truly general form for delays is considered in dynamic problems (beyond simple discrete delays or exponential kernels as it is usually done), a very large range of qualitative properties for the dynamics can be encountered, including limit cycles. This theoretical contribution finds significant applications in the design and calibration of environmental policies but could also be applied beyond this field to various other economic dynamic systems exhibiting similar delays (advertising, capital building, ...).

Section 2 presents the optimal stock pollution control model with general delays. Section 3 is devoted to the study of the dynamics: we write the dynamics as an MFDE and we analyze the properties of the characteristic equations. We then derive properties in terms of optimal trajectories, studying the impact of the various parameters involved in the economic model. Section 4 concludes.

2. Introducing distributed delays in the standard pollution control model

We consider the introduction of delays in a standard dynamic partial equilibrium model including a representative producer/polluter and the environmental damage sustained by society. The standard social planner problem is

$$\max_{p(.)} \int_0^\infty [f(p(t)) - D(c(t))] e^{-\rho t} dt \tag{1}$$

[1] Another strand of literature uses the Hamilton–Jacobi–Bellman equation to solve delayed models for which a closed form can be obtained (Federico et al., 2010).

where $f(p(t))$ is the private benefit derived from the emissions $p(t)$, $D(c(t))$ is the environmental damage caused by the pollution stock $c(t)$ and ρ is the social discount rate with $\rho \in]0, 1[$. f and D have the standard properties of the literature: f positive, non-decreasing, concave, defined over $\mathbb{R}^+$ and respecting the Inada conditions and D increasing, convex and such that $D(0) = 0$.

We consider a general expression of the pollution accumulation process that allows for various forms of pollution diffusion. The accumulation equation can be written, with $\alpha > 0$ being the natural decay rate of pollution, as

$$\dot{c}(t) = -\alpha c(t) + \theta \int_{t-\tau_2}^{t-\tau_1} p(u)\, \mu(t-u)\, du, \tag{2}$$

where $0 \le \tau_1 < \tau_2 < \infty$ and $\mu(.)$ is a probability density function on $[\tau_1, \tau_2]$ such that $\int_{\tau_1}^{\tau_2} \mu(u)du = 1$.

θ ($\theta \in [0, 1[$) is the "technological" factor reflecting the portion of the pollution generated by the economic activity that will leak into the environment.[2] This factor thus depends on local productive and environmental conditions. The cleaner the production process or the better calibrated the fertilizer application, the lower θ.

Expression (2) is convenient to embrace a wide range of distribution structures. The choice of function $\mu(.)$ will depend on the specific accumulation process of the problem considered. Pollution emitted at time t will be released in several weighted loads, distributed *a priori* across a time interval $[t + \tau_1, t + \tau_2]$. More precisely, pollution emitted at time t will reach the stock at time $t + u$, with weight $\mu(t + u)$, where $u \in [\tau_1, \tau_2]$. Expressing the delayed accumulation process $\mu(s)ds$ as a probability density function allows us to encompass a large range of complex pollution problems characterized by significant site specific heterogeneity in the pollutants velocity (Gaines and Gaines, 1994) that convert a spatial heterogeneity into a temporal one.

Before solving the general problem, let us give a few examples of possible distribution specifications. It is clear that if $\mu(v) = \delta_\tau(v)$, where the delay τ is strictly positive and $\tau \in [\tau_1, \tau_2)$, the dynamic reflects the standard homogeneously delayed accumulation process such as is found in Winkler (2008). Another relevant example is

$$\mu(v) = \frac{e^{-\vartheta v}}{\int_{\tau_1}^{\tau_2} e^{-\vartheta s} ds} \quad \text{for } v \in [\tau_1, \tau_2].$$

Here the parameter ϑ sets the repartition of the pollution load in time: the higher ϑ the earlier the emissions reach the stock within the time interval $[\tau_1, \tau_2]$.

3. Optimal pollution control with distributed delays

In order to highlight the specificities of our results with distributed delays, let us briefly recall the salient properties of the optimal pollution control problem with discrete delays such as they have been characterized by Winkler (2008, 2011). Under the assumption of a separable utility function such as the one in (1), the core properties of the benchmark optimal control model are preserved in the presence of discrete delays. The latter operates a mere "translation" of the steady state and of the corresponding optimal path towards a higher pollution stock. The saddle path property, the monotonicity of the optimal path and even the finite dimension of the stable manifold are maintained. Furthermore, the longer the delay the higher the pollution stock and the emission level at the steady state.

[2] In the case of nitrate contamination θ is tantamount to the portion of the fertilizers applied that are not assimilated by the crops and other local organisms. For polluting gases it corresponds to the portion of gas produced that has not been captured by end of pipe abatement devices.